DOCUMENT RESUME

ED 272 367 SE 046 695

TITLE	Mathematics 7-8. Minimum Standards Leadership Series 1985.
SPONS AGENCY	Ohio State Dept. of Education, Columbus. Div. of Elementary and Secondary Education.
PUB DATE	Feb 86
NOTE	100p.
AVAILABLE FROM	Ohio Department of Education, Division of Elementary and Secondary Education, 65 South Front Street, Columbus, OH 43215.
PUB TYPE	Guides - General (050)
EDRS PRICE	MF01/PC04 Plus Postage.
DESCRIPTORS	Algebra; *Cognitive Objectives; Computation; Computers; *Curriculum Development; *Grade 7; *Grade 8; Instructional Improvement; Junior High Schools; Learning; Learning Disabilities; Mathematics Anxiety; *Mathematics Education; Objectives; Secondary Education; *Secondary School Mathematics; Sex Bias; Time on Task

ABSTRACT

Guidelines concerning the development, supervision, and implementation of seventh and eighth grade mathematics curricula are presented in this document. Section one gives a brief overview of the grades K through six curriculum, providing terminal objectives for grade six. Section two describes traditional tasks in the curriculum for grades seven and eight and discusses Algebra I. The third section addresses specific curriculum components: (1) variables and literal expressions; (2) equations and inequalities; (3) geometry; (4) measurement; (5) probability and combinations; (6) problem solving; (7) applications; (8) functions and relations; (9) statistics; (10) technology; and (11) numbers and numeration. Each component is discussed, with specific objectives and sample problems and activities. The fourth section identifies and discusses questions related to specific and crucial issues in mathematics education. They are: (1) time on task; (2) calculators and computers; (3) mathematics anxiety; (4) sex equity; (5) ability grouping; (6) learning disabilities; and (7) teacher preparation. Also, the Second International Study of Mathematics is reviewed. The appendices contain resource material for each of the issue areas except mathematics anxiety, and also the solution to problems given as examples in section three. (JM)

****
* Reproductions supplied by EDRS are the best that can be made *
* from the original document. *
****

ED272367

MINIMUM STANDARDS LEADERSHIP SERIES 1985
OHIO DEPARTMENT OF EDUCATION

MATHEMATICS
7-8

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)

- ☑ This document has been reproduced as received from the person or organization originating it.
- ☐ Minor changes have been made to improve reproduction quality.

- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS MATERIAL HAS BEEN GRANTED BY
Mary J. Poston

TO THE EDUCATIONAL RESOURCES INFORMATION CENTER (ERIC)."

SE 040 695

STATE BOARD OF EDUCATION

Wayne E. Shaffer, President ..... **Bryan**
Patricia Smith, Vice President ..... **Worthington**
Jean E. Bender ..... **Akron**
Paul Brickner ..... **Willoughby**
Richard C. Glowacki ..... **Toledo**
Mary Goodrich ..... **Zanesville**
Jack C. Hunter ..... **Youngstown**
Virginia E. Jacobs ..... **Lima**
Robert H. Johnson ..... **Springfield**
Mary R. Lindner ..... **Cincinnati**
William E. Moore ..... **Woodsfield**
Sue Ann Norton ..... **Westerville**
Paul F. Pfeiffer ..... **Massillon**
Ronald M. Popek ..... **Cleveland**
C. J. Prentiss ..... **Cleveland**
Edwin C. Price, Jr. ..... **Cincinnati**
Chester A. Roush ..... **Kettering**
Anthony J. Russo ..... **Mayfield Village**
Sally R. Southard ..... **Oxford**
Jo A. Thatcher ..... **Portsmouth**
Karen L. Worley ..... **Elyria**

Franklin B. Walter
Superintendent of Public Instruction

STEERING COMMITTEE

G. Robert Bowers
Assistant Superintendent

Irene G. Bandy
Assistant Superintendent

Nancy Ann Eberhart
Director
Division of Inservice Education

James W. Miller
Director
Division of Educational Services

Robert L. Moore
Assistant Superintendent

Mary J. Poston
Director
Division of Elementary and Secondary Education

DEVELOPMENT COMMITTEE

Margaret B. Kasten, Chair
Mathematics Consultant, K-12
Division of Elementary and Secondary Education

Ruth F. Earls
Health Education Consultant K-12
Division of Elementary and Secondary Education

Joe Logsdon
Industrial Arts Consultant
Division of Elementary and Secondary Education

Carol Reigard
Consultant
Division of Educational Services

Floyd F. Shipman
Consultant
Division of Elementary and Secondary Education

Debby Waddell
Consultant
Division of Special Education

FOREWORD

Ohio's educators have traditionally worked hard to provide the best possible education for their students. The activity to improve education in Ohio during the past several years has been remarkable. Public and nonpublic schools have made great strides in implementing the Minimum Standards for Elementary and Secondary Schools.

Numerous teachers and administrators have commented on the utility of the Ohio Department of Education's implementation series consisting of fourteen documents that have been available since 1983. These publications have been found to be a great help in designing educational programs that not only meet the requirements of standards, but enable schools to consider various ways in which to provide quality education.

As educators continue to seek to provide excellence throughout their programs, there have been many inquiries about other parts of the standards not addressed by the implementation series. The department has identified areas of most common concern and has prepared this leadership series consisting of nine documents. As was the case with most of the documents in the original implementation series, the leadership series serves to specify the requirements of standards. Additional information, guidelines, procedural steps, and sample forms may be presented that go beyond what the standards require. The purpose is to provide information to help schools plan to meet their needs and provide the best possible education for their students.

The assistance of school personnel who contributed their time, expertise, and materials is greatly appreciated. Recognition and sincere thanks are extended to the department staff members who worked to make these publications possible.

Franklin B. Walter

Franklin B. Walter
Superintendent of Public Instruction

TABLE OF CONTENTS

INTRODUCTION 4

THE MATHEMATICS CURRICULUM 5

KINDERGARTEN THROUGH GRADE SIX MATHEMATICS CURRICULUM 6

- General Description 6
- Sixth Grade Terminal Skills 7

TRADITIONAL MATHEMATICS PROGRAMS IN THE SEVENTH AND EIGHTH GRADES 10

- The Seventh and Eighth Grade Curriculum as Review and Remediation 10
- The Seventh and Eighth Grade Curriculum as Preparation for Algebra 11
- The Seventh and Eighth Grade Curriculum as it Ought to Be 11
- Algebra I in Eighth Grade 12

ACHIEVING EXCELLENCE IN SEVENTH AND EIGHTH GRADE MATHEMATICS 13

- General Description 13
- Instruction 13
- Learning Outcomes 18
  - Variables and Literal Expressions 19
  - Equations and Inequalities 21
  - Geometry 24
  - Measurement 28
  - Probability and Combinations 32
  - Problem Solving 36
  - Applications 40
  - Functions and Relations 42
  - Statistics 45
  - Technology 49
  - Numbers and Numeration 50

CONSIDERATIONS FOR CURRICULUM DEVELOPERS 52

- Time on Task and Academic Learning Time 52
- Calculators and Computers 54
- Math Anxiety 55
- Sex Equity in Mathematics 56
- Mathematically Able Students and Ability Grouping 57
- Learning Disabilities 59
- Teacher Preparation for the Middle Grades 61
- Second International Study of Mathematics Achievement 61

TABLE OF CONTENTS

SUMMARY 66
RESOURCES 67
Appendix A: Time On Task 71
Appendix B: Calculators and Computers 74
Appendix C: Sex Equity in Mathematics 77
Appendix D: Mathematically Able Students and Ability Grouping 79
Appendix E: Learning Disabilities 80
Appendix F: Teacher Preparation for the Middle Grades 86
Appendix G: Solutions to Problems 89

INTRODUCTION

This document is intended as a resource for those individuals concerned with the development, supervision, and implementation of the seventh and eighth grade mathematics curriculum. The document is divided into four main sections. The first section provides a brief overview of the kindergarten through sixth grade mathematics curriculum. Terminal objectives for sixth graders are provided.

The second section gives a brief description and overview of the traditional tasks to be accomplished in the seventh and eighth grade mathematics curriculum. Additionally, Algebra I in eighth grade is discussed.

The third section addresses specific components of the seventh and eighth grade mathematics curriculum. After the general description and comments regarding appropriate instruction, learning outcomes are enumerated. The curriculum is categorized using eleven strands: variables and literal expressions, equations and inequalities, geometry, measurement, probability and combinations, problem solving, applications, relations and functions, statistics, technology, and numbers and numeration. The importance of each strand is discussed briefly, followed by specific objectives and sample problems and activities appropriate for seventh and eighth graders.

In the final section, questions related to some specific and crucial issues in mathematics curriculum development are identified and discussed. Those issues are as follow: time on task and academic learning time, calculators and computers, math anxiety, sex equity in mathematics, mathematically able students and ability grouping, learning disabilities, and teacher preparation for the middle grades. Additionally, the fourth section contains a description and discussion of the concerns raised in the Second International Study of Mathematics.

It is not anticipated that the reading audience will find all parts of the document equally useful. For example, it may be that classroom teachers will use the sample problems and activities more often than they study the results of the international study. Similarly, curriculum supervisors may not be as interested in the specific objective statements as in the discussion relative to the uses that might be made of computers. Those responsible for hiring teachers of seventh and eighth grade mathematics may be especially interested in the section concerned with teacher preparation. In total, the document identifies and addresses issues and concerns affecting seventh and eighth grade mathematics programs that should be helpful to individuals in a variety of roles.

There is general agreement that the seventh and eighth grade mathematics curriculum is an area of concern and dissatisfaction. It is hoped that this document will serve as a catalyst and resource for those who can and will improve mathematics for students in Ohio.

MATHEMATICS CURRICULUM

THE MATHEMATICS CURRICULUM

Mathematics is the chief problem-solving tool for our society.
(Hill, 1981, p. 6)

Mathematics is often called a survival skill. In fact, individuals rarely die because of lack of adeptness with mathematics. However, while mathematical proficiency may not be necessary to survive, it is certainly necessary to thrive. The scientific and technological society of tomorrow will require a high level of mathematical skill. The number of uses and areas of application for mathematics is exploding. Mathematics itself is changing in scope and definition. The school curriculum should respond to the need for increased mathematical proficiency. This document sets forth a curriculum for the crucial middle years of the seventh and eighth grades. The suggested curriculum will help meet the needs of students in a technological society.

GENERAL DESCRIPTION

KINDERGARTEN THROUGH GRADE SIX MATHEMATICS

Prior to any consideration of the seventh and eighth grade curriculum, a comprehensive review of the kindergarten through sixth grade curriculum should be completed. While differences exist in emphases and content in elementary mathematics across Ohio and the nation, there are general observations and suggestions that do apply.

GENERAL DESCRIPTION

Typically, the elementary students in Ohio, like those in most other states, do well on whole number computation. *Ohio Assessment Results* (n.d.) indicates that most students can add, subtract, multiply and divide. Ninety-two percent of fourth graders could correctly add: 534 + 268. This fact should not be disregarded. There is a generally held perception that students can't add. Yet by fourth grade, the vast majority have attained skill in addition with regrouping.

A review panel of educators, associated with the Ohio assessment, predicted student performance on the addition objectives at 77.88 percent. The panel's desired performance for students was 89.29 percent. In reality, students' performance was 92.75 percent. Students do very well in addition.

Fourth grade performance on the division objectives was satisfactory as well. At least 70 percent of the students performed the following divisions accurately:

$3\overline{)69}$	75% correct
$3\overline{)219}$	72% correct
$4\overline{)288}$	70% correct

Yet, when these same students were asked to transfer division skills to solve a problem they had minimal success. For example, only 19 percent of the students could solve the following problem:

> You and three friends decide to share a pizza. It costs $3.60. How much must each person pay to share the cost equally?

This problem was presented as a multiple choice item with five alternative responses. At chance level 20 percent of the students should have correctly answered the item.

SIXTH GRADE SKILLS

The examples presented illustrate the primary concern with the existing mathematics curriculum. There has been an overemphasis on certain topics, such as computation, to the detriment of key topics such as measurement, geometry, estimation, statistics, problem solving and applications. This narrow focus has continued into the seventh and eighth grades. For the proposed seventh and eighth grade curriculum to be implemented successfully, instruction in K-6 will need to focus on problem solving and to use an expanded definition of basic skills.

SIXTH GRADE TERMINAL SKILLS

The success of the proposed seventh and eighth grade curriculum will depend in large part on the K-6 mathematics curriculum which precedes it. For this reason it is desirable to list terminal objectives for sixth grade students. This list is neither a list of minimum competencies nor a statement of all the important mathematical concepts taught in grades kindergarten through six. These are "baseline" skills for grades kindergarten through six. Teachers of seventh and eighth grade students should appropriately consider this content as mastered and proceed accordingly. Students who have not mastered this content should be remediated, but students, parents, and teachers should be aware that the content is remedial. While the content listed below should not be taught *per se* in seventh and eighth grades, it will be necessary to use the content in problem-solving and application situations to maintain skill levels.

Numbers and Numeration. The student is able to

- recognize the place value of digits in numbers through millions
- recognize the place value of digits in numbers through hundred-thousandths
- write a number in fraction or decimal notation given a rational number in the other form
- factor whole numbers into primes and find the greater common factor and least common multiple of numbers
- round numbers to the nearest thousand, hundred, ten, one, tenth, and hundredth
- use a calculator to do calculations with whole numbers and decimals when a calculator is most appropriate

SIXTH GRADE SKILLS

Operations and Properties. The student is able to

- perform the four basic operations on whole numbers
  - a. addition (four-digit numbers)
  - b. subtraction (four-digit numbers)
  - c. multiplication (three-digit numbers)
  - d. division (two-digit divisor, four-digit dividend)
- use a standard algorithm for each of the four basic operations on "reasonable" common fractions, decimals, and mixed numbers
- find the square and cube of whole numbers less than 30
- find the square root of whole numbers less than 400 to the nearest whole number
- compare and order fractions, decimals, whole numbers, and integers
- apply mathematical properties (e.g., commutative, associative, distributive, order of operations, equivalent expressions) in meaningful ways

Geometry. The student is able to

- recognize, sketch, and use the defining characteristics of horizontal lines, vertical lines, parallel lines, perpendicular lines, and intersecting lines
- classify and use the defining characteristics of simple plane and solid figures
- identify and explain similarity and congruence
- locate points on a coordinate plane
- name and sketch prisms, pyramids, cones, and spheres, and identify defining characteristics

Measurement. The student is able to

- select and use appropriate units to measure length, area, volume, weight, capacity, time, temperature, and money
- use formulas to compute areas and perimeters of common polygons and circles
- convert from one common unit of measure to another within the same measurement system
- use common metric prefixes

SIXTH GRADE SKILLS

Problem Solving. The student is able to

- write a mathematical expression for a word phrase
- draw upon a variety of strategies in solving mathematical problems (e.g., make an appropriate sketch, make a table, look for a pattern)
- solve traditional one- or two-step problems presented in worded format
- identify irrelevant information in solving a problem and determine when given information is insufficient for a solution
- use graphs, charts, or tables to identify needed information, identify patterns, note trends, and draw conclusions

Probability and Statistics. The student is able to

- read, interpret, and construct tables, charts, and graphs
- find the mean, median, mode, and range for a set of data

Estimation and Approximation. The student is able to

- execute approximate calculations by rounding numbers
- use simple techniques for estimating quantity, length, distance, and weight
- determine when a particular result is precise enough for a specified purpose
- perform appropriate mental computations with two-digit whole numbers

Attitudes. The student is able to

- give examples of how frequently mathematics is used by individuals in their daily lives
- work in a consistent and persistent manner when solving mathematical problems and exercises
- exhibit confidence in being able to apply personal skills and knowledge to attack and solve problems
- work independently on problems or exercises for a reasonable time before seeking assistance

TRADITIONAL PROGRAMS

TRADITIONAL MATHEMATICS PROGRAMS IN THE SEVENTH AND EIGHTH GRADES

The mathematics curriculum of the seventh and eighth grades has been historically, and continues to be, an area of concern. Smith (as cited in Wheeler and Bezuszka, 1982, p. 4) has referred to mathematics in the middle years as a "...barren curriculum...a review and minor extension of the mathematics of grades one through six." Traditionally, there have been two tasks to be accomplished in these years:

- remediation of areas in the K-6 curriculum that were not mastered
- preparation for algebra

Both goals are noble and appropriate: however, these goals are not always attained, and they may limit the focus of a comprehensive mathematics curriculum.

THE SEVENTH AND EIGHTH GRADE CURRICULUM AS REVIEW AND REMEDIATION

Theoretically, the spiral curriculum provides careful development of concepts and topics over a period of years. Students are introduced to a topic, such as place value, in first or second grade. The topic is touched upon at progressively more difficult levels throughout the elementary curriculum. In practice many teachers seem to be unable to build upon previous learning, and consequently reintroduce topics at a low level. Typically, during the seventh or eighth grade almost the entire curriculum is retaught. Two points should be noted about this approach. First, the volume of material in the mathematics curriculum is so great that no single topic is covered with any depth or intensity. Second, many students become frustrated or bored. Typically, there are two kinds of students in seventh and eighth grade mathematics: those who have already mastered most of the content being covered and those who have mastered very little of the content. Often students in the first group, who are the potential mathematicians and scientists, are frustrated and bored in mathematics classes and seldom rekindle an interest in mathematics. Students in the second group may well need reteaching of some concepts, but the "survey" approach used does not provide the variation or depth of instruction and practice needed. This group also becomes frustrated and bored in mathematics courses and will probably stop taking mathematics as soon as possible.

SEVENTH AND EIGHTH GRADE CURRICULUM

THE SEVENTH AND EIGHTH GRADE CURRICULUM AS PREPARATION FOR ALGEBRA

The curriculum during the seventh and eighth grades attempts to prepare students for algebra by formalizing and extending work done with variables and equations. Unfortunately, preparation for algebra is often undertaken with inadequate attention to the learner's ability to deal with abstract concepts. Many adolescents are just entering the period in their conceptual development when they will be able to deal with abstractions such as variables and equations. Therefore, such concepts need to be introduced cautiously and linked carefully to concrete concepts.

THE SEVENTH AND EIGHTH GRADE CURRICULUM AS IT OUGHT TO BE

The seventh and eighth grade curriculum *should* be used to solidify understandings and skills initially developed in earlier grades. Remediation, if necessary, should not be just a brief review of topics that have been mystifying and confusing students for years. Planners of remediation should consider the characteristics of the students, learning theory, and mathematical content in establishing in-depth instruction for seventh and eighth grade students. The curriculum should also contain significant "new" mathematical concepts and provide a foundation for understandings and skills to be developed in the high school mathematics curriculum.

ALGEBRA I

ALGEBRA I IN THE EIGHTH GRADE

No examination of the seventh and eighth grade mathematics curriculum would be complete without consideration of what is commonly called "Algebra in Eighth Grade." Many schools in Ohio provide the option of eighth grade Algebra I. Several implicit assumptions are in operation whenever a select group of students is allowed or encouraged to take Algebra I in eighth grade. The first assumption is that there is little or nothing that these students will miss by not taking the standard eighth grade mathematics curriculum. This alone should cause decision makers to give thoughtful consideration to the standard eighth grade curriculum. The second implicit assumption is that the more mathematically able students will benefit by acceleration because they will be able to take five years of "high school" mathematics . Unfortunately, examination of enrollment records sometimes indicates that a high percentage of accelerated students drop out of mathematics at the end of the sophomore or junior year. They, therefore, have no mathematics for the last year or two of high school. If district personnel determine that a significant percentage of the students accelerated in eighth grade do not complete a five-year sequence, they should examine selection criteria. In addition, the high school program in which these students were enrolled should be reviewed. If a student does not complete the five-year sequence, it may well have been better to have delayed his/her introduction to high school mathematics until ninth grade.

Algebra I in eighth grade is a correct program option for some Ohio students. Selection of these students must be done carefully with attention to the needs of each individual student and to the appropriateness of the entire K-12 mathematics program.

All of the above observations and comments are predicated on the assumption that a district has in place and is working with a traditional mathematics sequence and curriculum. There are curricula, under development and in place, which integrate algebraic content for all students beginning in seventh or eighth grade. These curricula, which do not attempt to differentiate between college-bound and noncollege-bound students, may promote a quality mathematics education for all students.

REMEDIAL INSTRUCTION

ACHIEVING EXCELLENCE IN SEVENTH AND EIGHTH GRADE MATHEMATICS

GENERAL DESCRIPTION

Seventh and eighth grade mathematics must continue to perform the historical functions: 1) remediation of elementary school mathematics and 2) preparation for algebra. In addition, mathematical content must include many topics that students have not previously encountered. These topics should take advantage of the awakening intellect of the adolescent. This time is crucial for building interest and excitement and for challenging students to become involved in mathematics. The seventh and eighth grades should serve as a bridge between the common curriculum given to all students in elementary school and the more specialized, elective curriculum available at high school. The mathematics curriculum should still be general in nature, that is, the content should span a variety of topics.

INSTRUCTION

Despite lip service often given by teachers to "hands-on, laboratory type teaching," evidence suggests that students in the seventh and eighth grade mathematics classes are most often taught using the "lecture-recitation" method found in colleges. Instructional strategies and techniques used with seventh and eighth grade students need careful consideration.

For Remedial Students: While the content for the vast majority of students should be more than review, some students will need remediation. It is important that this remediation utilize alternative instructional strategies. Students who did not attain a concept the first or second or third time it was taught, will probably not attain the concept by doing it the **same way—one more time—louder.** Many samples of alternative instructional strategies can be found in the Ohio Department of Education document entitled *Intervention* (1983).

> The following intervention strategies are not meant to be prescriptive. They are but a few examples of the myriad of alternative strategies that could be used to support a district's intervention program. These mathematics strategies do not represent the scope and sequence of a mathematics program, but rather illustrate that the instruction/intervention process should be provided K-12. (*Intervention*, 1983, p. 38)

REMEDIAL INSTRUCTION

The following is a selected sample from the intervention document.

Suppose the objective is to teach an elementary algebra class how to square a binomial, e.g., $(x + 3)^2 = x + 6x + 9$. One of the most persistent mistakes students make is squaring each term only:

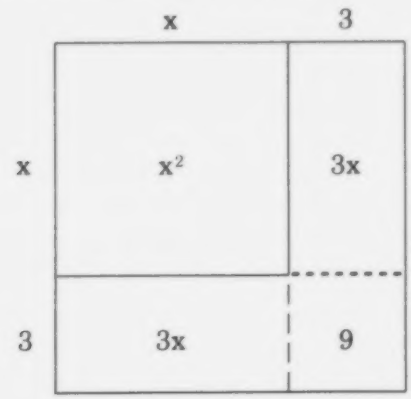
$$(x + 3)^2 = x^2 + 9 \text{ (incorrect)}$$

Research suggests that understanding is promoted when more than one way of viewing an idea is presented. Therefore, we might employ an axiomatic method:

(A)

$(x + 3)^2 = (x + 3)(x + 3)$	by definition
$= x(x + 3) + 3(x + 3)$	distributive property
$= x^2 + x\,3 + 3\,x + 9$	distributive property
$= x^2 + 6x + 9$	collect like terms

(B) We could introduce a geometric model, treating $x + 3$ as the side of a square and $(x + 3)^2$ as its area.



(C) Or we could introduce "George," a mnemonic device to assist students' recall of the various products to be found.

$$(x + 3)^2 = x^2 + 6x + 9$$

(Intervention, 1983, p. 38)

INSTRUCTION FOR AVERAGE STUDENTS

For average and above average students: Despite the fact that many of these students may not need remediation and some have attained the Piagetian stage of formal operations, there is still the need for a developmental approach to teaching. Students are not ready for the formal, definitional approach often used. Concepts should be "developed" rather than "presented."

The focus of mathematics in the seventh and eighth grades should be problem solving. The methods, content, and general classroom environment should support and encourage problem solving. Meiring (1980) has addressed at length problem solving in intermediate and junior high instruction:

> Intermediate and junior high school students need... problem-solving experiences which provide some other dimensions to their problem-solving development. By this age, they are ready to give more specific thought to other kinds of formal processes and techniques which they are using in problem-solving situations. Some classroom experiences can be profitably directed toward an awareness of the various stages in solving a problem. We again caution, however, against any kind of insistence that students employ a 4-step or other model to their problem-solving efforts. We encourage that attention be given to the importance of certain activities in the problem-solving process: use of strategies, awareness of reasoning procedures, attention to problem organization, recognition of attitudinal elements of problem solving, management of personal resources. Students need more opportunities with problem situations that arise out of their own experience or which have open-ended or investigation aspects. Formulating and developing their own problems is another activity which should be encouraged. Problems arising from the students' backgrounds, particularly ones which require the collection and use of data, are valuable for transfer of problem-solving skills from the classroom. Other problem situations may be selected which require the application of specific content to practical situations calling upon interdisciplinary skills.
>
> In general, efforts should be made to engage students with problems which are more sophisticated and lengthy. This might entail a grouping of related problems embedded in one problem setting, or a single problem which has several easily identifiable subgoals that are necessary to achieve a more important overall goal. A guided discovery lesson can be designed according to this second format. Either approach can be useful in planning for ability differences within a group.

INSTRUCTION FOR AVERAGE STUDENTS

Students should begin to assume more responsibility for their problem-solving efforts, expanding their tolerance for frustration and looking to each other and themselves for interpretation and evaluation of results. They need to begin to engage in active hypothesis development, conjecture testing, generalizing, inductive and deductive reasoning, and independent learning. It is desirable for students to change their perceptions of the teacher from that of dispenser of knowledge and answer evaluator to one of a source of interesting ideas, challenger of thinking, and experienced resource person.

Though students are engaging in more adult-like problem activities, they must also be recognized as being in a developmental stage and still requiring many considerations that we might associate with a younger learner. Many need assistance in knowing how to organize their work for a problem, how to construct a table, how to collect and record data. Some will still be at the level of learning to separate their work so that it does not all run together, preventing them or others from seeing relationships among the data.

A variety of materials ranging from odds and ends for manipulatives to sophisticated devices such as calculators, trundle wheels, and balances should be available. Sometimes it is mistakenly assumed that less reliance should be placed upon concrete objects as students grow older, that mathematics should become more of a mental and paper and pencil experience. Too often there is a tendency to rush this process. Students continue to need to use manipulatives, but those which they employ should gradually increase in sophistication. Consumables such as graph paper, string, and construction materials should be readily available at all times, and students should be encouraged to make frequent use of them.

Small and large group problem solving should receive more emphasis in these grades. Attitudes important to problem solving need nurturing, such as flexibility in choosing productive problem attacks. Reinforcement for alternative, creative, or unusual methods of solution should occur frequently. Occasional problem situations emphasizing competitive aspects of group problem solving can be used. The quality of solution rather than the speed of solution might be an appropriate evaluative criteria.

INSTRUCTION FOR AVERAGE STUDENTS

Problem solving in these grades should be more than an activity which is restricted to certain periods each week. It should be a teaching philosophy at all times. Problem situations can motivate and precede the development of algorithmic processes, for example, while supplying students with the concrete examples which will enrich the meaning behind the algorithms. The perceptive teacher will be alert to seizing student suggestions for problem investigations whenever they occur. Since such suggestions are likely to involve extensions of content material being studied, it will give students a sense of involvement and satisfaction that they are helping to shape the mathematical experiences they are enjoying.

One of the primary objectives for problem solving during the intermediate and junior high grades should be to provide a variety of problem-solving situations for study. Some of the kinds of situations which should receive emphasis include the following:

- Problems with superfluous or insufficient information.
- Problems that arise from the student's environment or interests.
- Problems which do not have well-defined answers or which involve inexact skills such as estimating.
- Problems posed in nonverbal settings utilizing manipulatives within the classroom or activities outside the classroom.
- Problems developed around central themes which are extended to substantive results but are subdivided into manageable parts.
- Problems requiring students to make choices about the degree of accuracy required or to be used within the problem.
- Multistep problems in which identification of intermediate step meanings are emphasized.
- Problems involving logic, reasoning, testing of conjectures, reasonability of information.
- Problems which require decision making to result from the mathematical outcomes.
- Problems which emphasize the practical application of mathematics to business or consumer situations.
- Problems requiring the collection and interpretation of data.
- Open-ended and investigatory problems.
- Problems involving modeling to explore interdisciplinary topics.

LEARNING OUTCOMES

- Problems which require students to conceptualize and make meaningful very large or very small numbers.
- Problems applying mathematics to school, home, or personal situations.
- Problem situations calling for more than one method of solution.

In selecting problems for student study, three criteria should be used: (1) the problem involves content that is appropriate for study, review, or reinforcement; (2) identifiable problem-solving skills or procedures are required for problem solution; (3) the student is required to do some original thinking in the solution of the problem. Using these three criteria and providing a good mixture of problems, ranging from real-life applications to purely student interest situations, will provide a rich and varied curriculum in problem solving...(Meiring, 1980, pp. 57-58)

LEARNING OUTCOMES

In addition to statements of learning outcomes, problems and activities are provided for each of the topic strands. The seventh and eighth grade mathematics outcomes will be categorized using the following strands:*

- Variables and Literal Expressions
- Equations and Inequalities
- Geometry
- Measurement
- Probability and Combinations
- Problem Solving
- Applications
- Relations and Functions
- Statistics
- Technology
- Numbers and Numeration

*Computation is not listed as a strand, because mastery of computation of whole numbers, fractions, and decimals is presupposed for those in the standard curriculum in seventh and eighth grades. If such mastery has not been achieved, students may require remediation. Computation will be reinforced and applied in the content strands. These outcomes represent a departure from the standard seventh and eighth grade curriculum. This alternative curriculum will better meet the needs of both college-bound and noncollege-bound students.

VARIABLES AND LITERAL EXPRESSIONS

Variables and Literal Expressions

The concept of variable is obviously important for students who plan to take algebra. It is also important for students who will not go to college. A study of variables should deal with the notion of replacement set and should not lead students to the mistaken idea that "a variable is a letter of the alphabet."

Objectives

The student will

- use variables to model physical or hypothetical situations
- apply standard algorithms to work with literal expressions

Problems: Variables and Literal Expressions

1. a. If you pay \$25 for an item and it goes up **u** dollars in price what will it then cost?
   b. What interpretation can be given when **u** is negative?*

2. a. If a gear is turned **x** notches, then **y** notches more, what is the total amount the gear has been turned?
   b. Call **x** positive when the turn is counterclockwise. Call **x** negative when the turn is clockwise. Does the answer to part (a) hold even when turns are in different directions?
   (MAA and NCTM, 1980, p. 73)

3. If the basic hourly pay rate for a machinist is \$8.65 and she is paid "time and a half" for hours worked in excess of 40 each week, write an expression for her gross pay in a week that she works **H** hours, where **H** might be equal to or greater than 40.
   (MAA and NCTM, 1980, p. 76)

4. A company has fixed costs of **b** dollars. Each item costs **a** dollars to make. How much would it cost to make **N** items?
   (MAA and NCTM, 1980, p. 76)

5. If you travel 120 miles in **h** hours what is your average speed?

*From Mathematical Association of America and the National Council of Teachers of Mathematics (1980, p. 73). Subsequent citations from this publication will be abbreviated MAA and NCTM.

VARIABLES AND LITERAL EXPRESSIONS

Activities: Variables and Literal Expressions

A. "Discover" π by comparing diameters and circumferences of round or cylindrical objects.

Object	distance around X	distance across Y	$\frac{X}{Y}$
water glass			
frisbee			
can			
tree			
choose another cylindrical object			

What can you conclude about the ratio of **X** to **Y**? Will this always be the case? How can you make sure?

B. Farmer Brown needs to fence a garden spot for her husband. She has 40 feet of chicken wire. Her husband wants a rectangular garden. Fill in the table and help Farmer Brown decide which dimensions will give Mr. Brown the greatest amount of garden space. (Use your calculator.)

Length	Width	Perimeter	Area
		40 ft	
		40 ft	
		40 ft	
		40 ft	
		40 ft	
		40 ft	
		40 ft	
		40 ft	

EQUATIONS AND INEQUALITIES

Equations and Inequalities

Study of equations and inequalities should build upon the work done with variables. The emphasis will be on mathematical sentences.

Objectives

The student will

- define and give examples of an open mathematical sentence
- interpret mathematical and/or number sentences that involve statements of equality, inequality or both
- develop a number sentence as one method of analysis of a word problem
- determine whether a given mathematical sentence (equality or inequality) is true, false or open
- solve and graph open sentences containing connectives "and" and "or" (e.g., $x < 2$ and $x + 3 = 10$; $5 > y$ or $2y = 4$)
- use variables in common formulas

Problems: Equations and Inequalities

1. There are three temperature scales in common use: Celsius (or Centigrade), Fahrenheit, and Kelvin. Conversions from Celsius temperatures to those measured on the Fahrenheit or Kelvin scale can be carried out by means of the following formulas:

$$F = \frac{9}{5}C + 32 \qquad K = C + 273$$

A chemistry textbook describes an experiment in which a reaction takes place at a temperature of 400° Kelvin. A student wishes to perform the experiment, but has only a Fahrenheit thermometer. At what Fahrenheit temperature will the reaction take place?

(MAA and NCTM, 1980, p. 89)

2. In many schools, for a grade of A in a course, 4 points are given for each credit unit in that course, 3 points for a B, 2 points for a C, 1 point for a D, and 0 points for an F.

   a) If a student has **a** credits with A grades, **b** credits with B grades, **c** credits with C grades, **d** credits with D grades and **e** credits with F grades, how many points are earned?

   b) What is the grade point average?

(MAA and NCTM, 1980, p. 77)

3. Suppose, for a given car, every mile of city driving uses 1/16 of a gallon of gas and every mile of country driving uses 1/22 of a gallon of gas. How far could this car go on 20 gallons of gas?

(MAA and NCTM, 1980, p. 98)

EQUATIONS AND INEQUALITIES

4. Two companies A and B offer you a sales position. Both jobs are essentially the same, but A pays a straight 8% commission and B pays $51 per week plus 5% commission. The best salespeople with either company rarely have sales greater than $4,000 in any week. For what amounts of sales will company A pay more money?

(MAA and NCTM, 1980, p. 82)

5. A 1984 Nova could be equipped with a diesel engine for about $2,500 more than it would cost with a standard gasoline engine. Average maintenance and fuel costs in 1984 for the diesel were about 12.7 cents a mile as compared with 27.1 cents a mile for the gasoline model. At these costs about how far would such a car need to be driven before the extra cost of the diesel version would be recovered?

(Adapted from MAA and NCTM, 1980, p. 83)

Activities: Equations and Inequalities

A. Whitman (1982) presents a method for developing intuitive equation solving skills. Her recommended activities, discussion about, and work with cross-number arrays, magic square, function machines, and equations with frames can be very helpful for students doing initial work with equations.

Fill the frames:

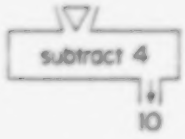
□ + 17 = 20
△ − 38 = 38
4 · ▽ = 80

□ + 3 = 8
3
+ ◇
10

4 + ▽ = 9
10 = 2 + ⬡

8		
6		
16	2	

Make the sum 30 in each direction.



Students should next try some examples such as these

a ⬡ + ⬡ = 16
b ▽ + ▽ = 15
c 1 = ◇ + ◇ + ◇
d 16 = □ · □
e ○ · ○ · ○ = 8
f □ + □ = □
g 5 + ◇ = 2 + ◇ + 3
h ⬡ + 3 = ⬡
i ◇ · ◇ = ◇

(Whitman, 1982, p. 201)

EQUATIONS AND INEQUALITIES

B. Obtain a balance and have students solve the following problem.

One of nine objects, identical in appearance, is slightly heavier than the rest. With only an equal arm insert scale balance available, how can the heavier object be determined in just two weighings?

(Meiring, 1980b, p. 60)

What extensions or variations are possible for this activity?

C. Use your calculator to build a table and answer the questions below. A storekeeper has a supply of 8¢ and 13¢ stamps. He regularly mails packages which require postage in varying amounts above 85¢, and he finds it possible to use the stamps to obtain the exact amount of postage.

a. Can every amount of postage above 85¢ be obtained exactly by using 8¢ and 13¢ stamps?

b. What is the smallest amount of postage which can be obtained in two different ways? (MAA and NCTM, 1980, p. 274)

Can you build a table for other denominations of stamps, say 20¢ and 13¢?

GEOMETRY

Geometry

Informal geometry, including spatial relations, and experience with two- and three-dimensional figures are important topics. Geometry should be taught with much use of manipulative materials, and it should be developed throughout the elementary mathematics curriculum. Geometry in the seventh and eighth grades should include an intuitive approach with an emphasis on spatial concepts and an introduction to formal geometry.

Objectives

The student will

- state and apply the Pythagorean theorem
- do basic compass and straight edge constructions
- sketch three-dimensional objects (tetrahedron, cube, cone, etc.)
- visualize transformations
- apply geometric concepts and skills to solve problems
- make an appropriate sketch for a given problem
- locate points on a coordinate plane
- identify parts of commonly studied geometric figures (e.g., diagonal, altitude, median)
- construct similar polygons (e.g., drawing to scale)

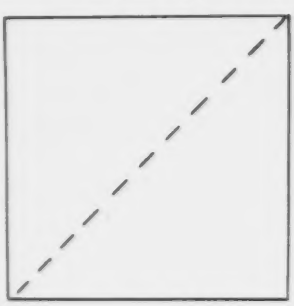
Problems: Geometry

1. a. What is the diameter of the largest circular table top (of negligible thickness) that could pass through a doorway 74 cm wide and 201 cm high?

   b. How many people could be seated around such a table?

   (MAA and NCTM, 1980, p. 176)

2. a. How much distance will a person save by cutting across a park that is one km on a side?



   b. How much time would the person save if a km can be walked in 12 minutes?

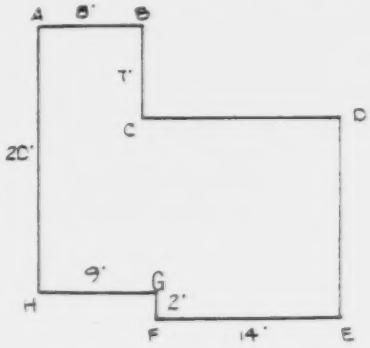
   (MAA and NCTM, 1980, p. 160)

GEOMETRY

3. After building the foundation of a small shed, you start laying sheets of plywood for the floor, and to your dismay one of the corners of one piece of plywood does not fit into a supposedly square corner of the foundation. By using the tape measure in your pocket and not the carpenter's square in the back of your pickup down the road, how do you decide who made the mistake—you in building the foundation or the sawmill in cutting the plywood?

(MAA and NCTM, 1980, p. 160)

4. The floor plan of a living room of a house is drawn here. There are many ways of describing the shape and size of this living room over the phone. One way could begin as follows: The room is in the shape of an octagon chart A, B, C, D, E, F, G, H, with all angles right angles. Letting each unit stand for a foot, consecutive vertices of this octagon are A = (0, 0), B = (8, 0), C = (8, -7),...



a. Finish the description of the living room by giving the coordinates of the other vertices of the octagon.

b. Is there a more convenient place to locate the x-and y-axes?

(MAA and NCTM, 1980, p. 167)

5. Carpeting is typically available in large rolls which come in the standard widths of 9 feet or 12 feet. You may buy any length you wish from such a roll; it is customary to advertise the price in dollars per square *yard*. A room you wish to carpet will often be an approximate rectangle **l x w** feet—you may have to cut out a small piece because of a door or a radiator, but this won't affect what you buy.

a. If you need to carpet a room, one of whose dimensions is either 9 feet or 12 feet, it's obvious from which size roll to buy. But suppose the room is 7 feet x 10 feet. Which roll will be cheaper assuming that each costs $6 per square yard?

b. Suppose the room is 7 ½ feet x 10 feet — which is cheaper then?

c. For what room sizes **l x w** is it cheaper to buy from the 9-foot roll? When do they cost the same?

d. Suppose the narrower 9-foot width is more expensive, say $8 per square yard. Is there any size room for which this is now cheaper?

(MAA and NCTM, 1980, p. 186)

GEOMETRY

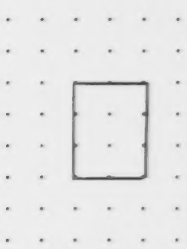
Activities: Geometry

A. There is an interesting relationship (called Euler's formula—V + F - E = 2) that exists between the number of faces, edges and vertices of polyhedron. Students can be led to discover the relationship by completing the following table. They should be supplied with physical models of the polyhedra.

Name	V	F	E	V+F-E
Tetrahedron				
Cube				
Octahedron				
Dodecahedron				
Icosahedron				

If a more challenging activity is desirable, omit the last column. Students should be encouraged to develop their own formula.

B. Geoboards and/or dot paper are useful materials for teaching geometric and measurement concepts. A geoboard may be purchased commercially or constructed using plywood and nails. One interesting activity involves the exploration or derivation of Pick's theorem.



Pick's theorem states the area of the polygon (vertices on dots or nails) equals the number of dots inside the polygon (**I**) plus one-half the number of nails or dots on the border (**B**) minus one.

$A = l \times w$
$A = 2 \times 3$
$A = 6$
$A = I + \frac{1}{2}(B) - 1$
$A = 2 + \frac{1}{2}(10) - 1$
$A = 6$

$A = I + \frac{1}{2} B - 1$

Students might:

1. Hunt for a counterexample using rectangles.
2. Verify the formula for polygons other than rectangles.
3. Try to determine other relationships between perimeter and area.

GEOMETRY

C. The following activity could be used as an introduction to transformational geometry.

1. Use the graph paper vertically. Put the origin in the center.

2. Locate these points:
   A = (0,0) D = (4,6)
   B = (5,10) E = (1,0)
   C = (5,4)

   Connect ABCDEA to make an arrow.

3. Multiply each coordinate in part 2 by -1 to get new coordinates:
   A_1 = (0, 0) D_1 = (,)
   B_1 = (-5, -10) E_1 = (,)
   C_1 = (,)
   Locate and connect these points. How is the new arrow related to the first one?

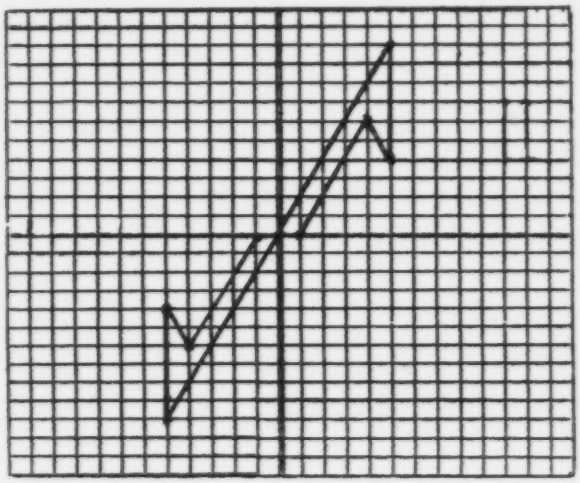
4. Start a new picture on another vertical piece of graph paper. Put the origin in the center.

5. Choose your own points on the graph paper. Use twelve or fewer.
   Record the coordinates here and name them.
   (,) (,) (,) (,)
   (,) (,) (,) (,)
   (,) (,) (,) (,)
   Locate and connect your points on the graph paper.

6. Multiply each coordinate in part 5 by -1 to get new points:
   (,) (,) (,) (,)
   (,) (,) (,) (,)
   (,) (,) (,) (,)
   Locate and connect your new points.

MEASUREMENT

7. Look carefully at your picture. Do you see that your original design has been turned? How has it been turned?



(Burger, 1982, p. 116)

Measurement

In addition to being an important topic in the development of mathematical literacy, measurement can be used as a motivational topic because of its "hands-on" nature. By the end of eighth grade, students should be able to make and use measures in the standard and metric systems. They should have some appreciation of the approximate nature of measurement and the concepts of standard error and error of measurement.

Objectives

The student will

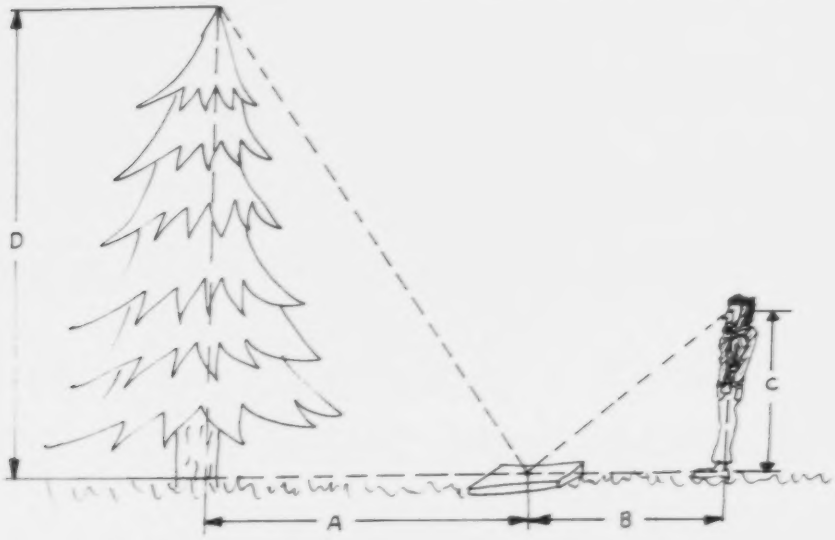
- state and apply area formulas for circle, rectangle and triangles
- state and apply volume formulas for cones, cylinder, spheres
- state and apply common formulas for surface area
- make judgments regarding accuracy and precision
- make reasonable estimates of measurement
- make indirect measurement
- select and use appropriate measurement units and tools
- measure accurately and effectively

MEASUREMENT

Problems: Measurement

1. You are an estimator for a paint supplier. Two spheres, each 65′ in outside diameter, contain compressed ammonia. The spheres are insulated with foam glass brick which must be painted with an asbestos asphalt aluminum coating. This coating goes on at the rate of 4 gallons per 100 square feet and costs $4.15 a gallon. What is your estimate of the cost of materials for coating the two tanks?
(MAA and NCTM, 1980, p. 193)

2. Estimating the height of an object using congruent triangles.



Step 1. Place a mirror on the ground at a convenient distance from the object. Since the mirror must be horizontal or level, more accuracy can be obtained if the mirror is placed on a two-or three-foot board which is then propped up with sticks or stones until level. Another way to obtain a level reflecting surface, without a mirror, is to use a small bowl of water, which has been painted black inside or has an inch of black dirt on the bottom. The water surface will be level even if the bowl is not sitting on level ground.

Step 2. Move back and forth until you are able to see the top of the object in the reflecting surface.

MEASUREMENT

Step 3. Make the following measurements:
a) object base to reflection of object top
b) reflection image to point directly under your eye
c) eye height

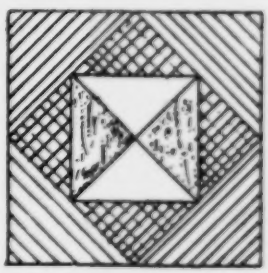
Step 4. Construct ratio and solve for **d.**

$$\frac{\mathbf{a}}{\mathbf{b}} :: \frac{\mathbf{d}}{\mathbf{c}}$$

a) Can "a" and "b" ever be equal?
b) Is this a reasonable method to use when measuring trees?
c) Can you think of better ways to measure tall trees?

3. A quilt is often made from square pieces. The pattern of one such piece, called "twelve triangles," is given at right. Suppose you want to make a 4-color quilt with the colors indicated by the shading at right. How much material will you need for each 40 cm square?

(MAA and NCTM, 1980, p. 185)

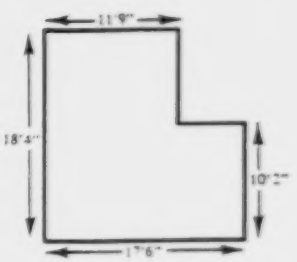


4. A sixth-grade class has decided to clean up the environment while learning how large one million is. Students will try to collect 1,000,000 twelve-ounce pop cans in their school. Determine how much space 1,000,000 cans will require.

(Meiring, 1980b, p. 61)

5. You have decided to carpet the living-dining room area of your home sketched at the right. You have not yet decided whether to use square foot sections or to order from a roll which is 12 feet wide. Determine the amount of each that you would need to order should you make that choice.

(Meiring, 1980b, p. 60)



MEASUREMENT

Activities: Measurement

A. Obtain several stopwatches and carefully instruct the children on their use and care. It may also be necessary to teach the children how to read the watches. Measure a course fifty meters long on the playground. Ask a child to run the course. Appoint someone as starter and have the other children in the group time the child's run. Mark the finish line so it is clearly visible to all the timers. When the child has run the course, record each of the times taken by the timers. If there is some variation, discuss what the children think are the reasons for the variation. Repeat the procedure using a different child to run the course. Help the group develop a method to minimize and handle any differences they find.

(Inskeep, 1976, pp 82-83)

B. Ask several children to measure the length of the sidewalk or fence around the school yard using metersticks, trundle wheels, or tape measures. Then ask them to compare their results and decide what to do with any differences. Which measure should be used? Should the measures be averaged? Suggest that they each remeasure the length and then compare their first reading with their second. Discuss any differences.

(Inskeep, 1976, pp. 82-83)

C. Find three medium large stones which you think are equal in size. Compare the "sizes" of the stones in as many ways as you can. Put them in order from smallest to largest. Were the stones equal in size in any of the ways you tried?

(MAA and NCTM, 1980, p. 190)

D. Find three containers of different shapes which you think have the same capacity.

   a. Find out whether you were right.

   b. Arrange the containers in order from smallest to largest capacity.

   c. Could you compare the containers in other ways?

(MAA & NCTM, 1980, p. 190)

E. The problem task is to be able to find the height of a pyramid from information which can be determined from the outside. Constructed within the classroom is a pyramid framework in which you can get inside. Take measurements, determine how the height can be found indirectly, and check your procedure by measuring.

(Meiring, 1980b, p. 59)

PROBABILITY AND COMBINATIONS

Probability and Combinations

In addition to being able to understand simple probabilistic statements contained in everyday situations like the weather forecast, students should have some basic understanding of basic probability concepts. Many of the decisions made in life, both large and small, can be based on probability. Understanding the basic concepts will empower students as they move into adulthood.

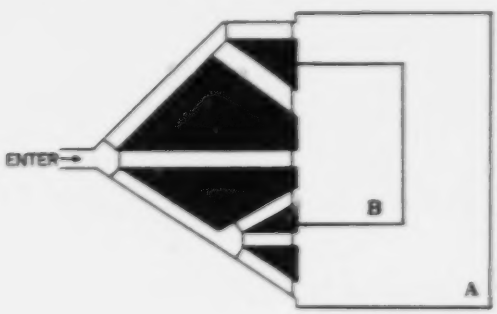
Objectives

The student will

- determine the probability of outcomes of common uncertain events (e.g., tossing a coin, rolling a die)
- make and interpret tree diagrams
- define a permutation as a distinct arrangement of a set of elements
- define a combination as a set of elements considered without regard to order

Problems: Probability and Combinations

1. (This problem is inspired by a popular short story, "The Lady or the Tiger?" by Frank Stockton.) The king and queen of a medieval kingdom arranged a marriage for their daughter to Prince Cuthbert from a neighboring kingdom. The princess accepted this plan without enthusiasm. A short time before the proposed wedding day, she met Reynaldo-handsome, clever, romantic, but only a peasant. Their love developed quickly and secretly, but inevitably the king learned of their relationship. Irate, he ordered that Reynaldo be thrown into a room full of tigers. But in response to his daughter's pleas, he offered a compromise: Reynaldo would walk through a maze, each path leading to one of two rooms. While the hungry tigers waited in one room, the hopeful princess should wait in the other room. If Reynaldo entered the latter room, he and the princess could marry.



The king showed the princess a map like this one of the maze and let her decide in which room to wait. Remember that Reynaldo does not have a copy of the map and can only guess which paths to follow. Which room is he more likely to enter, A or B?

(Armstrong, 1981, p. 136)

PROBABILITY AND COMBINATIONS

2. There are three cards in a hat. One card is white on both faces, one is red on both faces, and the third has one red face and one white face. The cards are mixed, and one is drawn from the hat and then placed on a table without showing the underside. If the face up is red, what are the odds that the other face is also red?

(Hill, 1974, p. 23)

3. The rules of the game Monopoly call for a player who rolls three consecutive doubles to go to jail. (A player who throws doubles, plays and throws again; if doubles are thrown a second time, the player plays and throws once more.) What is the probability that a player who is out of jail will go to jail for this reason on any turn?

(MAA and NCTM, 1980, p. 254)

4. A rough approximation to recent experience in the United States is that about 36% of all males and 40% of all females live to the age of 80.
   a. What is the probability that all four grandparents of a person will reach the age of 80?
   b. What is the probability that exactly 3 or 4 of a person's grandparents will live to the age of 80?
   c. What is the probability that a pair of grandparents on the same side will both live to the age of 80?
   d. What is the probability that no grandparent will live to the age of 80?
   e. What is the probability that at least one grandparent will live to the age of 80?
   f. Why do the answers in a) through e) not add to 1.00?

(MAA and NCTM, 1980, p. 252)

5. Of the students in the fifth grade in 1962, 76% graduated from high school in 1970; 40% of the high school graduates entered college in the fall of 1970; and half of those who entered college received a bachelor's degree in 1974.
   a. What is the probability that a randomly selected student in the fifth grade in 1962 entered college in the fall of 1970?
   b. What is the probability that a randomly selected student who was in the fifth grade in 1962 and graduated from high school in 1970 received a bachelor's degree in 1974?

(MAA and NCTM, 1980, p. 251)

PROBABILITY AND COMBINATIONS

6. Christmas tree lights in the past were connected in series; that is, if one light blew, they all went out. In such a string of a dozen lights, suppose that there is a 95% probability that a given light will last through the Christmas season.
   a. What is the probability that all the lights will remain on?
   b. Suppose now that twelve lights with the same reliability (probability of not burning out) are connected in parallel, so that whether any one stays on is independent of whether the others burn out. What is the probability that at least nine lights will still be burning at the end of the season?

(MAA and NCTM, 1980, p. 253)

Activities: Probability and Combinations

A. In social studies, we studied the effects of two-children families versus three-children families in population education. Your task is to create a model of two-child family growth versus three-child family growth. Assume that two couples, age 20, without children, are placed upon the island of Fertilia. These couples and all of their descendants are committed to having only two children per family. At the same time, two other couples, also age 20 and without children, are placed upon the neighboring island of Greater Fertilia. They and their descendants agree to have exactly three children per family. Show how the population will grow on these two islands over a period of 160 years. Make whatever assumptions you wish about when children are born, when people die, and so forth. But remember that all people must be counted and people must die as well as be born.

(Meiring, 1980b, p. 61)

B. During the 1961 baseball season, two players on the same American League team appeared to have a chance, after two-thirds of the season was completed, of breaking the old record of sixty home runs in a single season. The record was held by Babe Ruth for many years. The two players were Mickey Mantle and Roger Maris of the New York Yankees. Their records at the end of 106 games are given below:

Player	Home Runs (HR)	At Bats(AB)	HR/A
Mantle	40	387	.1034
Maris	41	414	.0990

PROBABILITY AND COMBINATIONS

Babe Ruth made his record when teams played 154 games a season. The 1961 season was the first in which the teams played an extended schedule of 162 games. Consequently, it was important to know what the chances were that either player would break the record in 154 games and what the chances were for 162 games. On the basis of the first 106 games, one may calculate the approximate number of at bats each player has left by determining the average number of at bats per game for each player. The result is 3.65 AB per game for Mantle and 3.91 for Maris. This leads to the following table of approximate at bats left, assuming each player plays all the remaining games.

	Number of AB Left	
Player	154 Games	162 Games
Mantle	176	204
Maris	188	219

The number of at bats left amounts to a series of **N** trials, each of which has only two possible outcomes—home run or no home run. If we assume independence and a constant chance of a given player hitting a home run on each trial, what we are concerned with is a binomial random variable for each player; **x** = number of home runs in **N** trials. On this basis, calculate the following:

1. The probability that Mantle will break the record
2. The probability that Maris will break the record
3. The probability that neither will break the record
4. The probability that at least one will break the record
5. The probability that both will break the record

Calculate all probabilities for 154-and 162-game schedules.

Players	154 Games	162 Games
Mantle		
Maris		
Neither		
1 or 2		
Both		

(Posten, 1981, p. 159)

C. One method of dealing with probability problems is to simulate the problem. Develop and describe a simulation for the following situation: Rich thinks he doesn't have to study for tests. He is willing to take his chances. Suppose he takes a true-false test in biology and doesn't know the answers to ten of the questions. What are his chances of getting sev or more of those ten questions correct by guessing?

(Travers, 1981, p. 216)

PROBLEM SOLVING

D. On the average, how often will two flipped coins come up heads? Three flipped coins come up heads? Four flipped coins come up heads? Can you **predict** how frequently five flipped coins will come up heads? How do the results for three coins coming up heads compare to a single coin being flipped and landing heads three times in succession?

(Meiring, 1980b, p. 61)

Problem Solving

Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-textbook problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unfearful of arriving at tentative conclusions, and they must be willing to subject these conclusions to scrutiny. (NCSM, 1977, p.2)

Objectives

The student will

- restate the problem in his or her own words
- identify assumptions being made
- illustrate given facts using Venn diagrams when appropriate
- analyze a problem and identify the additional information needed
- work in a consistent and persistent manner when solving mathematical problems and exercises
- exhibit confidence in being able to apply personal skills and knowledge to attack and solve problems
- write a mathematical expression for a word phrase
- draw upon a variety of strategies in solving mathematical problems (e.g., make an appropriate sketch, make a table, look for a pattern)
- solve traditional one- and two-step story problems

PROBLEM SOLVING

Problems: Problem Solving

1. For the celebration of the 300th anniversary of our nation, Oscar Gomez wants to form a line of people holding hands that stretches from New York to Los Angeles. The mayor of New York will hand a tricentennial message to the first person of the line who will hand it to the second person and so on until the last person hands the message to the mayor of Los Angeles. How long will it take to hand deliver the message?

   (Meiring, 1980, p. 58)

2. A father left 17 cows to his three sons, with instructions that the first son get one-half, the second son, one-third, and the youngest one, one-ninth of the cows. This, however, would have resulted in fractions of a cow, which would be of no use to anybody. How could they follow the father's instructions?

   A friend then offered to give them his one cow, making a total of 18. So it was possible to give the first son, one-half (9) of the cows, the second son, one-third (6), and the youngest son, one-ninth (2).

   Since 9 + 6 + 2 = 17, it was possible to return the loaned cow.

   Can you explain the "mystery" in the case of the cows?

   (Hill, 1974, p. 4)

3. A publishing company is preparing a manuscript for publication and, of course, wants it to be as free of errors as possible. The company hires two proofreaders, call them A and B, who read the manuscript independently. Proofreader A finds 36 errors, and proofreader B finds 32 errors. The publisher finds that included in these totals are 24 errors that were found by both proofreaders. What estimate can he make of the number of errors still unfound? The answer to this question will help him decide if it is worthwhile to hire a third proofreader.

   (Hill, 1974, p. 22)

Activities: Problem Solving

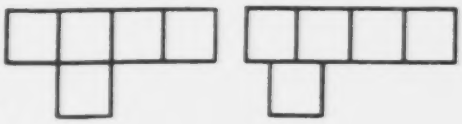
A. Conduct a poll of students in your school by classes to determine their favorite movie seen this year. Organize your information into some kind of graph which best conveys the point you wish to make. Make a list of conclusions which you feel are justified on the basis of your information. Make another list of related items that you feel that your information **does not answer.**

PROBLEM SOLVING

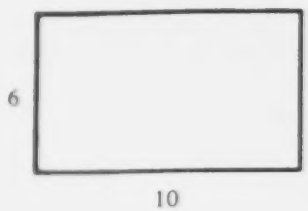
B. Students should be given quarter-inch or centimeter graph paper. They may also need scissors to complete the activities.

Pentomino Puzzlers

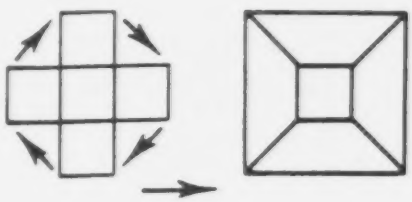
a. Find all the shapes possible that can be formed by joining five squares along an edge (pentominos). There are twelve.



b. Arrange the twelve pentomino shapes into a 6 x 10 frame; a 5 x 12 frame; a 4 x 15 frame.



c. Some of these shapes will make boxes without tops. Others will not. Find the ones which make boxes.



d. An employee at the box factory ordered sheets that are 4 x 5 from which to make topless boxes. How can you cut one of these sheets to get four boxes? Can you find more than one way?

(Meiring, 1980b, p. 59)

C. The five dart boards shown here are all squares with perimeters of 48 units. In each board, the circles are congruent and tangent to each other and to the edges of the boards.

Darts are thrown without aiming, and so each dart has an equal chance of hitting anywhere on the board.

PROBLEM SOLVING

1. If a game scores one point for each dart that hits inside a circle, which board would you rather play on?
2. If you throw 100 darts, how many points do you expect to score?
3. If a game consists of 100 points, how many darts would you expect to throw?
4. If you get +1 point for each dart inside a circle and -2 points for each dart outside a circle, what score do you expect after 200 throws? How many throws do you expect to need to score 100 points?
5. If a dart hits the dot in the center of a circle, you score +100 points, but if it hits any point on the circumference of a circle, you score -10 points. Under these conditions, which board would you choose to play on?
6. In a game for three players, scoring is as follows:

   Player A: +1 for each dart inside a circle
   0 for each dart outside a circle

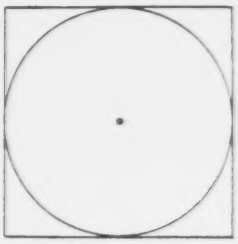
   Player B: +3 for each dart outside a circle
   0 for each dart inside a circle

   Player C: +2 for each dart inside a circle
   -4 for each dart outside a circle

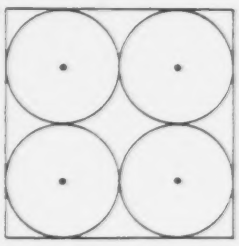
   Which player do you expect to be the winner?

   Does your answer depend on which dart board you choose?
7. Make up a set of scoring rules for a fair game for two persons.

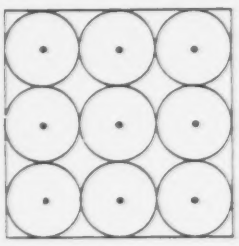
A



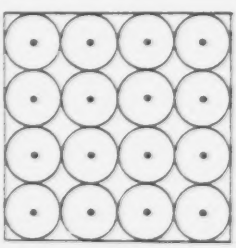
B



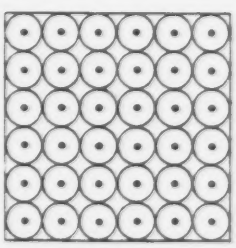
C



D



E



(House, Wallace, Johnson, 1983, p. 14)

APPLICATIONS

Applications

In seventh and eighth grades, nearly all mathematics topics can be applied. Applications can appropriately be made at the time the content, for example, statistics or variables, is studied. But in addition, students should make applications of mathematics which cross content strands and which do not directly follow a particular content concept. An applications strand adds emphasis and helps to assure that applications are not limited to the direct and trivial. Applications of mathematics should be made to situations that are of interest to the adolescent. Figuring wages, taxes, and household budgets may be appropriate for students at this level, but it is very important to include situations that directly impact the lives of middle-grade students. Such situations might involve sports, music, or clothing purchases.

Objectives

The student will

- apply integer arithmetic to solve problems
- apply fraction and decimal arithmetic to solve problems
- apply ratio and proportion concepts to solve problems
- apply percent concepts to solve problems

Problems: Applications

1. Following is the won-lost record of both divisions of the National League on September 18, 1985. Arrange the teams in correct descending order (within divisions) of "percentages" (without calculating percentages).

East Division	W	L	West Division	W	L
Chicago	70	79	Atlanta	61	88
Montreal	78	72	Cincinnati	81	67
New York	91	59	Houston	77	73
Philadelphia	71	77	Los Angelas	88	62
Pittsburgh	51	97	San Diego	75	75
St. Louis	94	56	San Francisco	59	91

If the divisional system had not been established, all twelve teams would be in direct competition for the National League Pennant. Arrange all twelve teams in descending order of "percentages" without calculating percentages. Can you make any judgment about which division is stronger?

APPLICATIONS

2. Your club will again handle concessions at the home football games this year. You need to place an order for food and drinks for the first game sales. Last season's total attendance for six home games was 13,593. Total food and drinks consumed amounted to 9,854 hot dogs, 6,512 hamburgers, and 21,678 soft drinks. This year's team is better, so attendance is expected to be at least 15% higher. But price hikes for food are expected to lower demand by about 5%. How many hotdogs, hamburgers, and soft drinks should you order for the first game?

(Meiring, 1980b, p. 60)

3. If Jesse Owens ran the hundred-yard dash in 9.6 seconds, what was his rate in miles per hour?

4. The average time for a bicycle to pass between the endpoints of a 100 m strip of sidewalk was 14.9 seconds. Determine this speed in km/h and label any intermediate results with units (e.g., km/min).

5. Suppose an eight-inch circular pizza serves one person.
   a. How many twelve-inch circular pizzas would be needed to serve a party of 18 people?
   b. Would the answer change if the pizzas were square shaped?

(MAA and NCTM, 1980, p. 183)

6. I want to build a 7′ diameter circular flower garden with a cement path around it 3′ wide and 4″ thick. What would the cement cost at $28.75 a cubic yard? (Be sure to draw a diagram.)

(MAA and NCTM, 1980, p. 193)

Activities: Applications

A. Alex wants to build a digital clock that operates on batteries, and he wonders whether it will take a car battery or flashlight batteries to operate it. Each digit is formed by a standard seven-segment display in which each segment is like a small elongated light bulb.
   a. Draw the segments which must be lit in order to produce each of the digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.
   b. How many segments are lit to display a zero? To display nothing? To display 3?

FUNCTIONS AND RELATIONS

Each segment uses 15 ma (milliamps, or thousandths of an amp) of current. Flashlight batteries (D-cells) are able to deliver about 2 amp-hours (2 amps for one hour, 2 ma for 1,000 hours, etc.).

c. For how long will D-cells keep a "four" lit? A "six"? A "two"?

d. A clock requires four displays to tell the hour and minute. For how long would D-cells keep a stopped clock reading 4:26? 12:46?

e. How many amp-hours are used by the first-digit display each day (i.e., the left one of the four)?

f. Do D-cells have enough reserve to keep a clock going for a long time?

g. Answer e) for the second digit.

h. How many amp-hours are needed to run the whole display for a day? How many for a year?

i. Would you recommend installing such a clock in a car?

(MAA and NCTM, 1980, p. 286)

Functions and Relations

Functions and relations are important in mathematics because they are "special" equations which furnish analytical descriptions of physical phenomena.

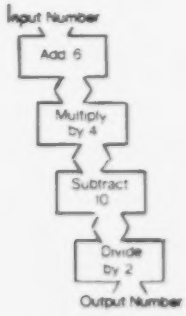
Objectives

The student will

- define and describe a relation
- define and give examples of functions
- represent a function in several ways (e.g., graph, mapping, set of ordered pairs, formula)
- define and use the absolute value function
- define and distinguish between relations and functions

Problems: Functions and Relations

1. Fill in the chart

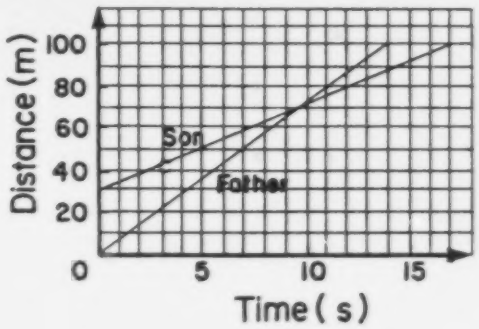


	Input Number	Output Number
a	4	
b	8	
c	12	
d		39
e		47
f		61

(Brannan, Schaaf, 1983, p. 42)

FUNCTIONS AND RELATIONS

2. A boy challenges his father to a 100 m race. The father allowed his son to start 30 m ahead of him. Their highly simplified graphs are shown in the figure.



a. How can you tell from the graphs that the father won the race by 3 s?

b. How far from the start did he overtake his son?

c. How long after the start was this?

(MAA and NCTM, 1980, p. 135)

3. A box for oranges might weigh about 2 kg when empty. A typical orange weighs .2 kg. What is the total weight if the box contains n oranges?

(MAA and NCTM, 1980, p. 76)

Activities: Functions and Relations

A. Doctors have found out that safe dosages of many drugs for humans can be determined from animal experiments if the same amount per unit of surface area is given to humans as to animals. Consequently, it is important to be able to estimate an individual's surface area. How might this be done?

(MAA and NCTM, 1980, p. 190)

FUNCTIONS AND RELATIONS

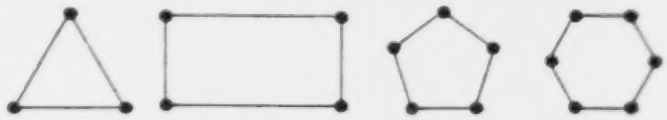
B.

UNIT 2 WORKSHEET
Pattern Solving

Name: _____ Date: _____ Score: _____

Complete each of the following exercises and try to write a formula for the *n*th term in the sequence that evolves.

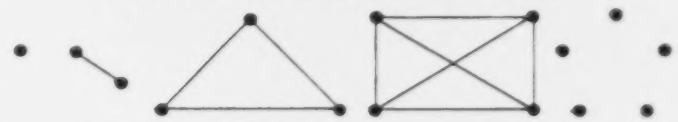
1. DIAGONALS OF A POLYGON (*n* = number of vertices)



MAXIMUM NUMBER OF DIAGONALS (*D*) THAT CAN BE DRAWN

n	3	4	5	6	7	... 50 ...	*n*
D							

2. POINTS AND LINES (*p* = number of points)

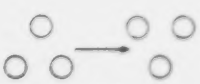


maximum number of lines (L) THE GIVEN POINTS DETERMINE

n	1	2	3	4	5	6	7	... 20 ...	*n*
L									

3. SHIFTING PENNIES

The minimum number of moves needed to *invert* a triangular arrangement of 3 pennies is one.



As more rows of pennies are added, the number of moves increases. Let *n* = number of rows in the triangle and *L* = number of moves to invert the triangle.

n	1	2	3	4	5	...	*n*
L							

(Bower, 1983, p. 62)

STATISTICS

Statistics

Statistics is an important topic in an information age. The ability to comprehend numerical information has become a basic skill. Anyone wishing to understand the graphs, surveys, and other numerical information presented in magazines and newspapers needs this skill. Nearly all forms of decision making rely on numerical information.

Objectives

The student will

- calculate the mean, median, mode, and range
- detect misuses of statistical or numerical information
- make logical inferences from statistical data (statistical inference)
- collect and organize data
- make meaningful interpretations of commonly used statistical terminology (i.e., mean, median, mode)
- conduct experiments involving counting and measuring

Problems: Statistics

1. Survey several seventh grade classrooms and several eighth grade classrooms. Determine how many students in each grade own home computers. Determine which brand is most popular. Is there a difference in the percentage of boys and girls who own computers? Is there a difference between the percentage of seventh graders and eighth graders who own computers? How can the information be organized? Can you make conjectures about computer owners from the data collected?

2. The broken-line graph in figure 1 shows how the proved natural gas reserves have changed over the last thirty years. The bar graphs show how gas production and additions to reserves have changed over the same period. Use the graphs to complete, for each year, columns 1 through 5 of table 1. (The table has been shortened to save space.)

   Use the data in column 5 of table 1 to construct a bar graph describing "Number of Years' Supply at Current Consumption." (In 1955 the supply of gas would have lasted 22 years, i.e., 220 : 10, at current consumption.)

   The period from 1955 to 1967 was the period of greatest addition to our natural gas reserves. Make an estimate of the average annual addition to reserves in those years by inspecting the graph. Compare your estimate with the result obtained by finding the average annual addition from 1955 to 1967 in the completed table 1.

STATISTICS

Figure 1 and table 1 show that 1968 was the first year in which the production of natural gas was greater than the increase in proved reserves. Production has exceeded reserve increases every year since then. Use the information in table 1 to find the average annual addition to reserves from 1968 to 1974. Find data to extend the graph to the present.

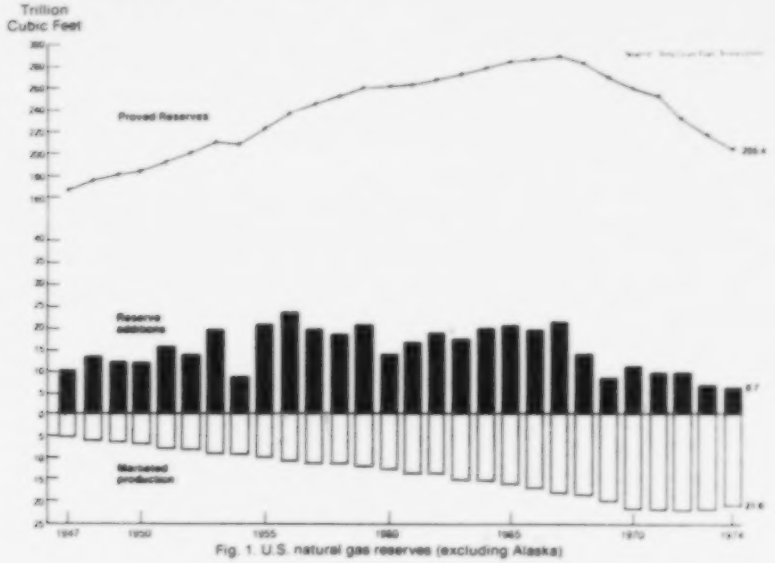


Fig. 1. U.S. natural gas reserves (excluding Alaska)

TABLE 1
Trends in U.S. Gas Production and Reserves
(In Trillions of Cubic Feet)

Year	(1) Additions to Reserve	(2) Production (Consumption)	(3) Difference between Additions and Productions (1) — (2)	(4) Proved Reserves	(5) Number of Years Supply of Gas Will Last (4) — (2)
1947					
1950		6		183	
1955	22	10	+12	220	22.0
1956	24	11	+13	233	21.2
1960	15	13	2	260	20
1967	22	18	+4	285	15.8
1968	13	19	-6	279	14.7
1973	9.0	22	-13	218	9.9
1974	8.7	21.6	-12.9	205.4	9.5

(Fishman, 1979, pp. 175)

The preceding problem uses data gathered from 1947 to 1974. Work the problem as it is presented. Gather data necessary to complete the charts through 1984. Were the conservation efforts of the late seventies successful? Does it appear that companies increased exploration efforts? Is there cause for concern regarding dwindling gas reserves?

3. At the eighth grade track meet, five students participated in the high jump. The first round jumps were 4′, 4′, 4′10″, 4′8″, and 4′11″. What are the mean, median, mode and range of the heights jumped on the first round?

Activities: Statistics

Pagni (1979) offers a variety of activities appropriate for various grade levels.

A. Dropping the meterstick and measuring the distance it falls before the subject can halt its movement can provide interesting data. Suppose each pair of students makes several trials, and the results are recorded and averaged (table 1).

Table 1
Record Of Meterstick Reaction Distances for One Pair

	Trial Number					
Student	1	2	3	4	5	Mean Distance
Judy	16 cm	13 cm	12 cm	11 cm	11 cm	12.6 cm
Mark	15 cm	14 cm	13 cm	14 cm	13 cm	13.2 cm

If the mean distances are assembled for all pairs, these values can be averaged to find the mean distance traveled for the class. The mean distances could also be displayed according to frequency distribution by classifying the results into intervals of stopping distances (table 2).

Table 2
A Possible Frequency Distribution For Mean Distances

Mean Distance	Frequency
20 - 24	1
15 - 19	3
10 - 14	4
5 - 9	2
0 - 4	0

In this example, four students reacted before the ruler fell more than fourteen centimeters but not before it fell ten centimeters. From the frequency distribution it is a simple matter to display the results in a histogram (figure 1). Examples of questions that can be posed with respect to this experiment are "What is the effect of practice on the reaction distance?" or "What is the range of (normal) human reaction distances?"

STATISTICS

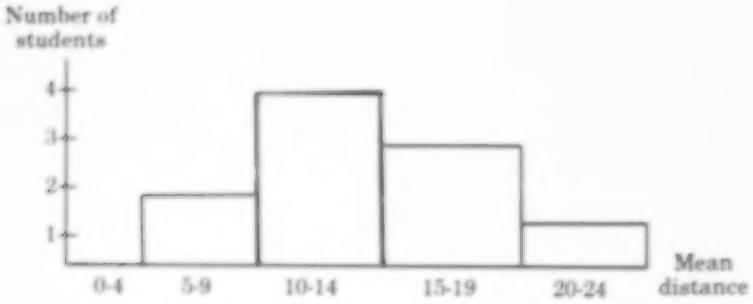
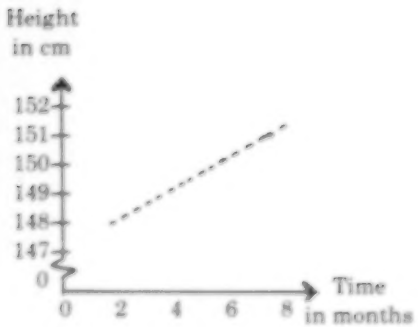


Figure 1.

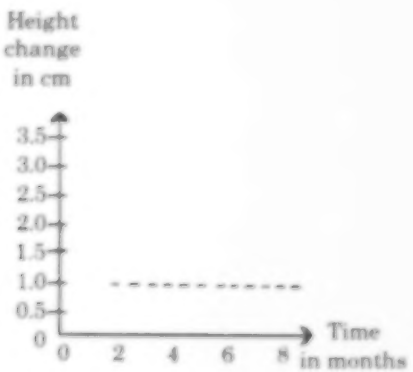
(Pagni, 1979, p. 56)

B. This example is introduced because it demonstrates a trait that can be graphed. If students measure their heights every two months, then the heights can be plotted against time (figure 2). To get an idea of growth rate, plot the change in height against time (figure 3).



Graph of heights

(Figure 2)



Graph of growth rate

(Figure 3)

This graph (figure 3) shows a steady growth rate, 1.0 centimeter every two months. If the student should "spurt" and grow more rapidly over a period of time, the growth-rate curve would not be horizontal.

(Pagni, 1979, p. 56)

TECHNOLOGY

Technology

Calculators and computers are increasingly important in the daily lives of adults. Most adults interact with these technologies in the workplace, as they go about their daily lives, and during recreation. If students are to reach their full potential, they must be prepared in school to interact with and effectively use technologies. Calculators should be available for all students in seventh and eighth grades, and they should receive instruction relative to the correct use of the calculator. Students in these years should also be introduced to computer literacy. Care must be taken that the time for computer literacy does not impinge on the time for mathematics instruction. Computers should be used as tools in the study of seventh and eighth grade mathematics.

Objectives

The student will

- use the computer as a tool
- use the calculator as a tool

Problems: Technology

1. (Computer) A Cryptarithm: HISS x HISS = GOSSIPY. This multiplication problem is coded; each letter stands for a single digit. Break the code and find the number that each letter represents.

   (Adapted from Roberts and Moore, 1984, p. 165)

2. (Computer or Calculator)
   Find at least three integers between -5 and +6 that will satisfy the equation $x^3 - 4^2 x - 17 x + 60$.

3. (Computer) Tickets to a basketball game are \$.75 for students and \$1.25 for adults. If 195 tickets were sold, producing receipts of \$201.25, how many of each type were sold?

   (Smith, 1984, p. 140)

4. (Calculator) Complete this table.

+			93
15		16	
	79		
	58		109

(Beardslee, 1978, p. 231)

TECHNOLOGY

Activities: Technology

A. *Variation of Nim* (2 players)

Materials: 1 Calculator

Procedures: The target number is 50. Players take turns adding any one-digit number to the display. The first player to obtain 50 in the display is the winner. Students should be encouraged to determine a winning strategy. Also, different target numbers should be used.

(Beardslee, 1978, p. 234)

B. Once upon a time, a group of students had the pleasure of attending a performance by Math E. Magic, a mathematically inclined magician. He involved them in several interesting mathematical magic tricks, the final one of which we want to share with you. He asked each member of the audience to choose a natural number and follow these instructions in order:

1. Write the number.
2. If your number is 1, stop.
3. If your number is even, divide it by 2 and then return to instruction 1 and follow through the sequence of steps with this new number.
4. If your number is odd, multiply it by 3, add 1, and then carry out instruction 1 with this new number.

When everyone had finished, he boasted that no matter what number is chosen, his instructions will always produce a 1. No one in the audience could find a counterexample to the magician's claim. Can you?

(Channell and Hirsch, 1984, p. 175)

Numbers and Numeration

Though the majority of "arithmetic" work should have been completed by the time a student enters seventh and eighth grades, there is a need to extend the addition, subtraction, multiplication, and division of whole numbers, fractions and decimals. Number topics such as exponents, roots and primes should be explored. The whole "number" area should be approached from a more formal viewpoint than was possible in earlier years.

NUMBERS AND NUMERATION

Objectives

The student will

- read and write any whole number
- state and use the definitions of rational numbers, irrational numbers, whole numbers, and integers
- state and use definitions of ratios, proportions, and percentages
- write numbers given in standard form using scientific notation and write numbers given in scientific notation in standard form
- write numbers in scientific notation and interpret numbers written in scientific notation
- use exponents
- use roots

Problems: Numbers and Numeration

1. If, as is commonly reported, the world population is doubling every 35 years, how much is it growing in half that time?

   (MAA and NCTM, 1980, p. 110)

2. Assume that in 1984 a liter of cola costs $1.00 and the inflation rate is 4 percent.
   a. How many years will it be before the cola will cost $2.00 a liter?
   b. If it were to cost $2.00 in 10 years, what would be the average inflation rate over the 10-year period?

3. An estate valued at $6,000 is to be divided among a wife, a son, and a daughter. Each is to receive $500, and the rest is to be divided so that the wife receives 3 times as much as the son, who receives as much as the daughter. How much will the wife, son and daughter each receive?

   (MAA and NCTM, 1980, p. 80)

Activities: Numbers and Numeration

A. Students should be asked to try to find the number of words in a set of encyclopedias in the school library.

   Things to consider:

   - encouraging groups using different strategies to compare results
   - finding a strategy that matches the need for accuracy

B. Is it feasible to collect 1,000,000 cans? How much space would be needed for storage? How many cans would each student need to bring each day of school? Plan such a project.

CONSIDERATIONS FOR CURRICULUM

CONSIDERATIONS FOR CURRICULUM DEVELOPERS

Those charged with the development of appropriate curriculum for seventh and eighth grade mathematics must consider the classical concerns in curriculum development: needs of student, needs of society, and integrity of the discipline. This section will identify and briefly discuss questions related to some specific and crucial issues in mathematics curriculum development and the Second International Study of Mathematics. Curricular issues are addressed in a question-answer format. Documentation, detailed ideas, and further discussion of some of the items may be found in the appendices.

TIME ON TASK AND ACADEMIC LEARNING TIME

WHAT IS TIME ON TASK?

Time on task is defined as the actual amount of time devoted to the learning process. Time on task may refer to teacher instruction, supervised student study or even specified homework related to the topic. (Further discussion of of this question is found in Appendix A.)

HOW IMPORTANT IN ACHIEVING OUTCOMES IS TIME AND THE WAY IT IS USED?

There are several important variables in the achievement of outcomes. These include learner ability, motivation, and attention. Time is also an important variable. It is a variable over which the teacher has a high degree of control. Research gives clear direction in terms of which activities are the most time effective. (Further discussion of this question is found in Appendix A.)

ACADEMIC LEARNING TIME

HOW CAN TEACHERS INCREASE THE AMOUNT OF TIME ON TASK FOR STUDENTS?

The following are some of the many techniques that a teacher may utilize to increase the amount of time on task:

- Developing and implementing a workable set of classroom rules and behaviors;
- Communicating a system of student responsibility and accountability which is perceived as fair and clear;
- Applying diagnostic and assessment procedures to determine the extent to which students possess the knowledge and skills necessary to learn a set of objectives as well as the ways in which students learn best;
- Helping students get ready to learn by specifying and repeating objectives;
- Providing students with plenty of opportunities for both guided and independent practice;
- Engaging in frequent and timely student encouragement, reinforcement and feedback; and
- Assigning, correcting and grading homework of appropriate difficulty, and returning homework assignments to students in a timely manner. (Connecticut Department of Education, 1984, p. 10)

WHAT IS ACADEMIC LEARNING TIME?

Academic learning time implies successful learning tasks which are neither too difficult nor too easy. Teachers can help all students to achieve a high rate of success by

- Teaching to the whole class initially, but using small group and individual instruction as needs develop for enrichment, extension or remediation;
- Providing for teacher/student interaction opportunities during the entire class lesson;
- Reteaching lesson content until students show they have learned it;
- Using regular reviews of key concepts and skills throughout the year to check on student retention.

Lengthening the school day or year, though significant, is not a sufficient condition in and of itself to increase student learning. The necessary accompaniments are effectively managing the use of time and improving the quality of instruction. (Connecticut Department of Education, 1984, p. 11)

CALCULATORS AND COMPUTERS

CALCULATORS AND COMPUTERS

WHAT IS THE APPROPRIATE ROLE OF CALCULATORS?

Seventh and eighth grade students should have access to calculators for virtually all activities. Students who are proficient in whole number computation can focus more readily on applying mathematics and solving problems when relieved of the drudgery of calculation. Students who have difficulty with whole number computation have often been unable even to attempt higher cognitive level tasks because of this deficiency. Use of the calculator will allow these students to attempt mathematical problems that are realistic, useful, and important. Nearly all adults who deal with whole number computation with any regularity use calculators. Mathematics education should prepare students for effective use of mathematics in adulthood, which implies use of calculators. (Further discussion of this question is found in Appendix B.)

WHAT IS THE APPROPRIATE ROLE OF COMPUTERS?

The mathematics program for seventh and eighth grade must take full advantage of computers. Where computers are available, they should be used extensively in seventh and eighth grade mathematics. Computers can be used in the management and delivery of mathematics instruction in seventh and eighth grades. They should be used in mathematics instruction when the topics or ideas being taught are taught best or most easily with the aid of the computer. (Further discussion of this question is found in Appendix B.)

IN WHAT WAYS MIGHT THE COMPUTER BE USED IN MATHEMATICS?

Computers can be used to do many of the things that are done in seventh and eighth grade mathematics. Some uses are more appropriate than others, and some uses are inappropriate. Computers can be appropriately used for drill and practice, tutorial activities, simulations, and problem-solving activities. Computers can be appropriately used with preprogrammed software, or students might be required to do simple programs to solve problems or investigate mathematical properties. (Further discussion of this question is found in Appendix B.)

MATH ANXIETY

MATH ANXIETY

WHAT IS MATH ANXIETY?

Math anxiety is a tension which interferes with the ability to solve mathematical problems. This tension impedes the concentration, attention, and memory needed for mathematics.

WHAT ARE THE EFFECTS OF MATH ANXIETY?

Individuals experiencing math anxiety tend to avoid situations involving mathematics in order to reduce their level of anxiety. One way to avoid math anxiety is not to enroll in mathematics courses. This in turn results in a restriction of career choices since mathematics acts as a "critical filter" that determines whether one can enter most of the higher-paying fields in which employment opportunities exist. (Jacobs, 1983)

WHO SUFFERS FROM MATH ANXIETY?

Math anxiety affects children and adults. More females are subject to it than males. A critical factor for children is the attitude of their parents and teachers toward mathematics, since the attitude of these individuals is a potent force in the development of children's attitudes.

WHAT CAN BE DONE TO REDUCE MATH ANXIETY?

The reduction of math anxiety should be a concern of all teachers of mathematics. The following techniques may help reduce math anxiety:

- Individualizing instruction
- Introducing games and labs
- Using concrete examples and hands-on techniques to introduce abstract concepts
- Making the classroom atmosphere one where any question is acceptable
- Teaching techniques for reducing stress and tension
- Teaching strategies for solving mathematical problems
- Assisting students to replace debilitating habits with self-instruction and self-encouragement
- Providing positive mathematics experiences

SEX EQUITY IN MATHEMATICS

SEX EQUITY IN MATHEMATICS

WHAT IS THE NATURE OF SEX DIFFERENCES IN MATHEMATICS PERFORMANCE?

Achievement in mathematics is approximately identical for boys and girls through the elementary years. Differences favoring boys are found during the secondary years. These achievement differences, which appear in the middle grade years and continue to grow through the high school years, occur even when girls have been enrolled in the same mathematics classes as boys. (Further discussion of this question is found in Appendix C.)

WHY DO FEMALES EXPERIENCE MORE MATH ANXIETY AND ENROLL IN FEWER MATHEMATICS COURSES THAN DO MALES?

Social factors seem to be the major influence on the preponderance of females who experience math anxiety. Mathematics activities are frequently stereotyped as being a male domain. During the middle school years, when girls become aware of this stereotyping of math as masculine, they begin avoiding mathematics. Other factors which may be pertinent include some of the following:

- Differential expectations of parents and teachers regarding mathematics performance of males and females
- Scarcity of female mathematicians as role models
- Early failure experiences in mathematics courses
- Fear of competition in a traditionally male discipline
- Sex role stereotypes in mathematics material
- A belief that mathematics is not as useful or important for females as for males

HOW CAN SEX EQUITY IN MATHEMATICS BE INCREASED?

If sex equity in mathematics is to be achieved, it must be consciously and actively pursued by teachers of mathematics. The seventh and eighth grade years are especially important. Activities which teachers might pursue include the following:

- Examine the career relevance of mathematics for women
- Invite to class guest speakers who can be positive role models of females in mathematics
- Use results of standardized career planning tests carefully
- Encourage mathematically talented females to act as effective peer role models

MATHEMATICALLY ABLE STUDENTS

- Examine textbooks for sex-fair treatment of males and females
- Encourage counselors to promote serious mathematics study among female students
- Locate resources related to the issue of sex equity in education
- Promote the acceptance of mathematical talent among female students
- Increase opportunities for successful mathematics experiences for females

MATHEMATICALLY ABLE STUDENTS AND ABILITY GROUPING

WHO ARE THE MATHEMATICALLY ABLE STUDENTS?

Mathematically able students are chiefly characterized by an ability to master the standard curriculum in less time than is needed by the majority of students (Further discussion of this question is found in Appendix D.)

WHAT DOES RESEARCH SAY ABOUT ABILITY GROUPING?

A review of the research relative to ability grouping in mathematics yields contradictory information. Therefore, when making decisions relative to ability grouping in mathematics, other criteria should be considered. (Further discussion of this question is found in Appendix D.)

WHAT PROGRAM EFFORTS HAVE BEEN MADE FOR THE MATHEMATICALLY ABLE?

Four major types of experiences have been found to be effective with mathematically able students.

1. Accelerated educational experiences including grade acceleration, advanced subject matter programs, early graduation, and concurrent enrollment at another school or training agency.
2. Special experiences such as topic seminars, independent study and research, mentorships, internships, and other intensive work with experts in a given field of study.
3. Guidance services including personal and social awareness, academic planning and performance, and vocational and career planning.
4. Organizational options, such as clustering, resource rooms, and self-contained classrooms. (Ohio Department of Education, *Rule for School Foundation Units for Gifted Children*, 1984)

OPTIONS

HOW HAVE OPTIONS BEEN DESIGNED?

Among the most common ways to organize options for mathematically able students are cluster grouping within the heterogeneous classroom, part-day special grouping of gifted students, special homogeneous classes, special schools for the gifted, peer tutoring, team teaching, individualized programmed instruction, Individual Guided Education (I.G.E.), mentor-assisted enrichment projects, and enrichment activities. (Correll, 1978)

WHAT ACTIVITIES OR TOPICS SHOULD BE STRESSED FOR THE MATHEMATICALLY ABLE STUDENT?

Although the following activities should be performed by all students, mathematically able students may engage in some of these activities at an earlier time and in greater depth than more typical mathematics students.

- Locating and processing quantitative information
- Collecting data
- Organizing and presenting data
- Interpreting data
- Drawing inferences and predicting from data
- Estimating measures
- Measuring use of appropriate tools
- Mentally estimating results of calculations
- Calculating with numbers rounded to one or two digits
- Using technological aids to calculate
- Using ratio and proportion to deal with rate problems in general and with percent problems in particular
- Using imagery, maps, sketches, and diagrams as aids to visualizing and conceptualizing a problem
- Using concrete representations and puzzles that aid in improving the perception of spatial relationships
- Using the concepts of sets and some of the language of sets
- Understanding elementary function concepts including dynamic models of increasing or decreasing phenomena
- Using algebraic symbolism and techniques
- Mastering highly specialized vocabulary (NCTM, 1980, p. 7)

LEARNING DISABILITIES

WHAT ACTIVITIES MIGHT BE DELETED OR RECEIVE LESS EMPHASIS FOR ABLE STUDENTS?

Less attention should be directed to the following activities for all students. Mathematically able students will need to spend even less time on these activities:

- Performing isolated drill with numbers apart from problem contexts
- Performing paper-and-pencil calculations with numbers of more than two digits
- Converting measures given in one system to corresponding measures in another system
- Working with tables whose usefulness as aids to calculation has been supplanted by calculators and other technological aids (NCTM, 1980, p. 7)

LEARNING DISABILITIES

WHAT IS A LEARNING DISABILITY?

A learning disability is a disorder in one or more of the basic psychological processes involved in understanding or in using language, spoken, or written, which may manifest itself in an imperfect ability to listen, think, speak, read, write, spell, or to do mathematical calculations (Ohio Department of Education, *Rules for the Education of Handicapped Children*, 1982, p. 12). (Further discussion of this question can be found in Appendix E).

HOW LONG HAVE LEARNING DISABILITIES BEEN RECOGNIZED?

Learning disorders have been recognized and studied since the end of the nineteenth century. Medical, psychological, and educational specialists, who have studied the causes and characteristics of students with learning disabilities, have suggested various adjustments in learning activities. However, only recently have students with learning disabilities become an issue for the public school. (Ludlow, n.d.)

LEGISLATION

HAS LEGISLATION MANDATED SPECIAL ATTENTION TO LEARNING DISABLED STUDENTS?

In 1963 the term, learning disabilities, was introduced to describe students who were diagnosed as having a specific learning problem. By 1975, it was recognized that there was a need for special education services for handicapped students, including those with learning disabilities. Hence, Public Law 94-142, the Education For All Handicapped Children Act, was introduced....(Ludlow, n.d.)...The law specifies that education must be provided in the least restrictive environment and that whenever possible, the handicapped child must be educated with nonhandicapped peers. This practice has been referred to as mainstreaming. (Hasazi, et al., 1979)

WHAT ARE SOME OF THE CONSEQUENCES OF MAINSTREAMING?

The principal consequence of mainstreaming is that classroom teachers of mathematics must broaden their instructional repertoire to include general practices and specific strategies that will meet the needs of handicapped students. (Further discussion of this question and other issues related to mainstreaming can be found in Appendix E.)

WHAT ARE SOME GENERAL PRACTICES AND SPECIFIC STRATEGIES FOR WORKING WITH HANDICAPPED STUDENTS?

There are two kinds of activities that the mathematics classroom teacher should consider when working with handicapped students. The first activity consists of using the consultative services of special education teachers and other specialists. Effective mainstreaming necessitates cooperation between specialists and classroom teachers. The second category of activities involves knowledge of and proficiency with specific instructional techniques which can be used when working with handicapped students. (Further discussion of this question and examples of specific intervention techniques can be found in Appendix E).

INTERNATIONAL STUDY

TEACHER PREPARATION FOR THE MIDDLE GRADES

WHAT WOULD BE APPROPRIATE TEACHER PREPARATION FOR TEACHERS OF SEVENTH AND EIGHTH GRADE MATHEMATICS?

To be an effective teacher of mathematics for the middle grades, one must be responsive not only to the tumultuous, transitory years of adolescence, but also to a wide range of mathematical abilities and requirements. The challenge for middle school mathematics teachers should be to deal with the remedial teaching of arithmetic of the elementary school as well as with the substantive teaching of more advanced mathematics, problem solving, computers, and algebra.

In the past, the ability to teach mathematics students in the middle grades was hampered by patterns of teacher preparation and certification. Far too little attention has been addressed to the cognitive functioning of adolescents and to the specific mathematical content appropriate to seventh and eighth grade students. Teacher preparation must include both a strong content emphasis and an emphasis upon the special physical, psychological and cognitive needs of the seventh or eighth grade student. (Further discussion of this question can be found in Appendix F.)

SHOULD TEACHERS OF MATHEMATICS HAVE SPECIAL PREPARATION FOR MEETING THE NEEDS OF GIFTED AND HANDICAPPED LEARNERS?

Gifted and handicapped learners are students with unique needs. These needs cannot be met by simply accelerating or decelerating the regular mathematics curriculum. For the gifted student, the mathematics program needs to include an emphasis on the higher-level thinking skills involved in problem solving. For the handicapped learner, on the other hand, abstract concepts must be demonstrated with concrete and multisensory methods. These methods are complex and numerous enough to require special teacher preparation.

SECOND INTERNATIONAL STUDY OF MATHEMATICS ACHIEVEMENT

In September 1984, a summary report of the United States' results from the *Second International Study of Mathematics* (1984) was released by the International Association for the Evaluation of Educational Achievement (IAEEA). This section is based on the report. The data and findings have important ramifications for those concerned with the seventh and eighth grade mathematics curriculum.

INTERNATIONAL STUDY

The study included three components: (1) the intended curriculum; (2) the implemented curriculum; and (3) the attained curriculum. The intended curriculum reflects the context presented in curriculum guides, courses of study, syllabi, and textbooks. The implemented curriculum is the curriculum as taught and interpreted by classroom teachers. The attained curriculum represents what students actually learned. The study sampled students and teachers from twenty-four countries. In the United States students and teachers from more than 600 mathematics classrooms took part. There were two populations of students included in the study—eighth graders and twelfth graders. This document focuses on the results of the eighth grade study.

Academic and attitudinal data were collected from approximately 7,500 eighth grade students during the 1981-82 school year. The study also included information collected from teachers on various topics including personal background, content presented, and methodology. (The report did not include any specific content from the intended curriculum as presented in formal curriculum documents).

CLASS STRUCTURE, STUDENTS, AND TEACHERS

An analysis of the data yielded no startling revelations. Mathematics classes at the eighth grade had a median size of 26 students, a median length of 50 minutes (with a range from 40 to 60 minutes), and a median of 145 clock hours of instruction. Of the 236 classes in the eighth grade sample, 155 were classified as typical, 24 as remedial, 26 as enriched and 31 as algebra.

Some of the identified teacher characteristics may surprise some individuals. Contrary to what might be suggested by the "critical teacher shortage," the study found:

> It can be stated, based on median data, that the typical eighth grade mathematics class has a teacher who is experienced and well trained. This teacher is about 37 years old with 13 years of teaching experience, 8 of which have been spent in eighth grade mathematics. (IAEEA, 1984, p. 12)

The teachers sampled estimated that their students typically spent 2.3 hours per week on mathematics homework. Approximately two-thirds of the teachers reported that calculators were never used or not allowed in class.

INTERNATIONAL STUDY

IMPLEMENTED CURRICULUM TOPICS AND METHODOLOGY

Table 1 indicates the approximate number of classes teachers expect to spend on selected topics.

Table 1
Median Anticipated Number of Periods for Selected Topics
For Each Class Type

	Median Number of Periods			
Topic	*Remedial*	*Typical*	*Enriched*	*Algebra*
Common Fractions	20	20	10	2
Decimal Fractions	20	15	10	1
Ratio and Proportion	10	10	6	4
Percentage	15	15	10	2
Measurement	10	12	10	1
Geometry	10	15	15	1
Formulas/Equations	2	20	26	50
Integers	10	15	10	8
Probability and Statistics	0	5	5	0

(IAEEA, 1984, p. 20)

When asked how certain mathematical topics are taught, teachers of "typical" classes showed consistency.

> The teaching of *fractions and decimals* by the eighth grade teachers tended to be symbolic and formal rather than intuitive or concrete. (p. 26)
>
> As with fractions, a formal approach interpreting one concept by reference to another dominated the teaching of *ratio, proportion and percent.* (p. 27)
>
> *Integers* were usually introduced through the number line and physical situations. However, when presenting the operations the teachers relied instead on giving rules. Three-quarters of the teachers emphasized solving linear equations by performing the same operations on both sides. Exploratory or intuitive methods such as trial and error or arithmetic reasoning were rarely used. The teaching of *Algebra*, like arithmetic, appears to be rule oriented and focused on symbol manipulation. (p. 27)
>
> The prevailing technique used for teaching *formulas* in general was to present a formula and explain the meaning of the terms. (p. 27)
>
> Introducing topics in *geometry* also focused on a statement of definition and properties rather than on informal explorations and the uses of students' intuitions. (p. 28)

INTERNATIONAL STUDY

The reported methodology is disheartening but not surprising. Much has been written about the characteristics of students in the middle grades and about effective teaching of mathematics. The rule-example method of teaching typically employed is *not* a good way to teach most of these students. But these are the methods that teachers report. It is not that teachers do not know better.

> Teachers' opinions about the teaching of geometry were notably at odds with their reported practices. They affirmed that an intuitive approach is most meaningful, that concrete models and aids should be used and that activities to improve spatial ability should be included. But in reality the most emphasized approach was a statement of definitions; the only aids extensively used by the majority of the teachers were the ruler and protractor; and spatial relations were taught in only 39 percent of the reporting classes. (IAEEA, 1984, p. 24)

It is clear that teachers of mathematics in seventh and eighth grades use teaching methods that are overly formal and rigorous. It is also clear that the vast majority of teachers of eighth grade mathematics in the United States teach topics which should have been learned in the K-6 curriculum. Ninety percent reported covering 2/5 + 3/8, while 100 percent reported covering 1054-865 and 162 x 45. Both the typical content and the methodology used to teach that content seem highly questionable.

ATTAINED CURRICULUM RESULTS

Through pretesting and posttesting, the study attempted to determine what mathematics was learned in five areas: arithmetic, algebra, geometry, measurement, probability and statistics.
Comparison of United States data with the data of other countries indicates that the students in the United States are "close to the international average in achievement." (IAEEA, 1984, p. 51)

Eighth Grade Comparative Achievement Means

	United States		International
Topics	Pre	Post	Post
Arithmetic	42	51	51
Algebra	32	43	43
Geometry	31	38	41
Statistics	53	57	55
Measurement	35	42	51

INTERNATIONAL STUDY

It is also noted that the United States students scored at the international median in arithmetic, algebra and statistics. "They score at the 25th percentile internationally in geometry and well below the 25th percentile in measurement. By any reasonable standard there is considerable cause for alarm in these comparisons." (IAEEA, 1984, p. 54) What should be done? The content of the seventh and eighth grade mathematics program must be changed and different methods of teaching the "new" content must be used.

65

SUMMARY

SUMMARY

- Mathematics curriculum in seventh and eighth grades is typically unsatisfactory.
- Curriculum reform is a complex, multidimensional phenomenon. However, this should not be used as an excuse by district personnel to do nothing about changing and refining the seventh and eighth grade mathematics curriculum.
- This document offers background information, suggestions, and ideas to those interested in providing a seventh and eighth grade mathematics curriculum which will meet student and society needs.

RESOURCES

RESOURCES

Armstrong, Richard D. "An Area Model for Solving Probability Problems," *Teaching Statistics and Probability*, 1981 Yearbook of the National Council of Teachers of Mathematics, edited by Albert P. Schulte. Reston, Va.: The Council, 1981.

Beardslee, Edward C. "Teaching Computational Skills with a Calculator," *Developing Computational Skills*, 1978 Yearbook of the National Council of Teachers of Mathematics, edited by Marilyn N. Suydam. Reston, Va.: The Council, 1978.

Bley, Nancy S., and Carol A. Thornton. "Help for Learning Disabled Students in the Mainstream," *Mathematics for the Middle Grades (5-9)*, 1982 Yearbook of the National Council of Teachers of Mathematics, edited by Linda Silvey. Reston, Va.: The Council, 1982.

Bower, Howard G. "Eureka! A Problem-Solving Course for the Pittsburgh School District," *The Agenda in Action*, 1983 Yearbook of the National Council of Teachers of Mathematics, edited by Gwen Skufelt. Reston, Va.: The Council, 1983.

Brannan, Richard and Oscar Schaaf. "An Instructional Approach to Problem Solving," *The Agenda in Action*, 1983 Yearbook of the National Council of Teachers of Mathematics, edited by Gwen Shufelt. Reston, Va.: The Council, 1983.

Burger, William F. "Graph Paper Geometry," *Mathematics for the Middle Grades (5-9)*, 1982 Yearbook of the National Council of Teachers of Mathematics, edited by Linda Silvey. Reston, Va.: The Council, 1982.

Channell, Dwayne E. and Christian R. Hirsch. "Computer Methods for Problem Solving in Secondary School Mathematics," *Computers in Mathematics Education*, 1984 Yearbook of the National Council of Teachers of Mathematics, edited by Viggo Hansen. Reston, Va.: The Council, 1984.

Connecticut Department of Education. *Research-Based School Improvement Practices*. Hartford, Conn.: Author, 1984.

Correll, Marsha M. "Teaching the Gifted and Talented." *Fastback 119* Bloomington, Ind.: The Phi Delta Kappa Educational Foundation, 1978.

Driscoll, Mark. "Elementary School Mathematics," *Research Within Reach: Elementary School Mathematics and Reading*, reprinted by Ohio Department of Education. Columbus, Ohio: The Ohio Department of Education, no date.

RESOURCES

Fennema, Elizabeth. "Girls and Mathematics: The Crucial Middle Grades," *Mathematics for the Middle Grades (5-9)*, 1982 Yearbook of the National Council of Teachers of Mathematics, edited by Linda Silvey. Reston, Va.: The Council, 1982.

Findley, Warren, and Mirian Bryan. *The Pros and Cons of Ability Grouping*. Bloomington, Ind.: The Phi Delta Kappa Educational Foundation, 1975.

Fishman, Joseph. "Developing Classroom Applications Based on Socioeconomic Problems," *Applications in School Mathematics*, 1979 Yearbook of the National Council of Teachers of Mathematics, edited by Linda Silvey, Reston, Va: The Council, 1979.

Gray, William A. *Challenging the Gifted and Talented Through Mentor-Assisted Enrichment Projects*. Bloomington, Ind.: The Phi Delta Kappa Educational Foundation, 1983.

Haertel, Geneva D. "Literature Review of Early Adolescence and Implications for Science Education Programming," *Early Adolescence: Perspectives and Recommendations*. Washington, D.C.: National Science Foundation, 1978.

Hasazi, Susan E., Paul D. Rice and Robert York. "Mainstreaming: Merging Regular and Special Education," *Fastback 124*. Bloomington, Ind.: The Phi Delta Kappa Educational Foundation, 1979.

Hill, Thomas. *Mathematical Challenges II: Plus Six*. Reston, Va.: National Council of Teachers of Mathematics, 1974.

Hill, Shirley A. "The Agenda for Action as a Potential Agent for Change in the Mathematics Curriculum," edited by Jack Price and J.D. Gawronski. Reston, Va.: The National Council of Teachers of Mathematics, 1981.

House, Peggy A., Martha L. Wallace and Mary A. Johnson. "Problem Solving as a Focus: How? When? Whose Responsibility?" *The Agenda in Action*, 1983 Yearbook of the National Council of Teachers of Mathematics, edited by Gwen Shufelt. Reston, Va.: The Council, 1983.

Inskeep, James E. "Teaching Measurement to Elementary School Children," *Measurement in School Mathematics*, 1976 Yearbook of the National Council of Teachers of Mathematics, edited by Doyal Nelson. Reston, Va.: The Council, 1976.

International Association for the Evaluation of Educational Achievement. *Second Study of Mathematics, Summary Report: United States*. Champaign, Ill.: U.S. National Coordinating Center, 1984.

Jacobs, Judith E. "Equity Through Mathematics: Everyone's Responsibility," *The Mathematics Teacher*, Vol. 76, 7 (1983) 462-463.

RESOURCES

Ludlow, Barbara L. "Teaching the Learning Disabled," *Fastback* 169. Bloomington, Ind.: The Phi Delta Kappa Educational Foundation, no date.

Mathematics Association of America (MAA) and the National Council of Teachers of Mathematics (NCTM). *A Sourcebook of Applications of School Mathematics.* Reston, Va.: The Council, 1980.

Mathematics Association of America. *Recommendations on the Mathematics Preparation of Teachers: Council on Undergraduate Preparation in Math.* Reston, Va.: The Mathematics Association of America, Committee on Undergraduate Programs in Mathematics, 1983.

Meiring, Steven P. *Problem Solving...A Basic Mathematics Goal: Becoming a Better Problem Solver.* Columbus, Ohio: Ohio Department of Education, 1980a.

Meiring, Steven P. *Problem Solving...A Basic Mathematics Goal: A Resource for Problem Solving.* Columbus, Ohio: Ohio Department of Education, 1980b.

National Council of Teachers of Mathematics. *An Agenda for Action: Recommendations for School Mathematics of the 1980's.* Reston, Va.: The Council, 1980.

National Council of Supervisors of Mathematics. *Position Paper on Basic Mathematical Skills.* Reston, Va.: The Council, 1977.

National Science Foundation. *Educating Americans for the 21st Century.* Washington, D.C.: National Science Foundation, 1983.

O'Brien, Thomas C. "Five Essays on Computers in Education," *Phi Delta Kappan,* Vol. 66, 2 (1983) 110-113.

Ohio Department of Education. *Microcomputer Planning Guide.* Columbus, Ohio: Ohio Department of Education, no date.

Ohio Department of Education. *Needs Assessment: Mathematics Grade 8.* Columbus, Ohio: Ohio Department of Education, no date.

Ohio Department of Education. *Intervention.* Columbus, Ohio: Ohio Department of Education, 1983.

Ohio Department of Education. *Rule For School Foundation Units For Gifted Children.* Columbus, Ohio: Ohio Department of Education, 1984.

Ohio Department of Education. *Rules for the Education of Handicapped Children.* Columbus, Ohio: Ohio Department of Education, 1982.

Pagni, David L. "Application in School Mathematics: Human Variability," Applications in School Mathematics, 1979 Yearbook of the National Council of Teachers of Mathematics, edited by Sidney Sharron. Reston, Va.: The Council, 1979.

RESOURCES

Peterson, Dale. "Nine Computer Issues that Educators Must Address," *Popular Computing*, Special Issue (1984) 10-18.

Posten, Harry O. "Evaluating Exact Binomial and Poisson Probabilities without Tables," *Teaching Statistics and Probability*, 1981 Yearbook of the National Council of Teachers of Mathematics, edited by Albert P. Shulte. Reston, Va.: The Council, 1981.

Roberts, Robert S. and Margaret L. Moore. "Programming to Learn Problem Solving," *Computers in Mathematics Education*, 1984 Yearbook of the National Council of Teachers of Mathematics, edited by Viggo P. Hansen. Reston, Va.: The Council, 1984.

Smith, Susan. "Microcomputers in the Middle School," *Computers in Mathematics Education*, 1984 Yearbook of the National Council of Teachers of Mathematics, edited by Viggo P. Hansen. Reston, Va.: The Council, 1984.

Travers, Kenneth J. "Using Monte Carlo Methods to Teach Probability and Statistics," *Teaching Statistics and Probability*, 1981 Yearbook of the National Council of Teachers of Mathematics, edited by Albert P. Shulte. Reston, Va.: The Council, 1981.

Trafton, Paul. "Assessing the Mathematics Curriculum Today," *Selected Issues in Mathematics Education*, National Society for the Study of Education, Series on Contemporary Educational Issues, edited by Mary Montgomery Linquist. Berkeley, Calif.: McCutchan Publishing, 1980.

Walker, Decker F. "Reflections on the Educational Potential and Limitations of Microcomputers." *Phi Delta Kappan*, Vol. 66, 2 (1983) 103-107.

Wheeler, Margaret M. and Stanley Bezuszka S.J. "Middle Grade Mathematics: An Overview," *Mathematics for the Middle Grades (5-9)*, 1982 Yearbook of the National Council of Teachers, edited by Linda Silvey. Reston, Va.: The Council, 1982.

Whitman, Betsy S. "Intuitive Equation-Solving Skills," *Mathematics for the Middle Grades (5-9)*, 1982 Yearbook of the National Council of Teachers of Mathematics, edited by Linda Silvey. Reston, Va.: The Council, 1982.

APPENDIX A

TIME ON TASK

WHAT IS TIME ON TASK?

The National Council of Teachers of Mathematics believes the amount of time allocated to learning mathematics in elementary school should be increased.

> It should range from a minimum of about five hours a week in primary grades to a minimum of about seven hours a week in the upper grades. Part of this time can be gained by a program that stresses the application of mathematical skills in other subjects, especially the sciences and social studies. (NCTM, 1980, p. 20)

HOW IMPORTANT IN ACHIEVING OUTCOMES IS TIME AND THE WAY IT IS USED?

Instructional time must be used wisely because learning is the result of both the time engaged in the learning task and the quality of that engagement. Time on task can effect how readily new learning is applied and the concepts that are retained. (NCTM, 1980)

> Time on task has always been considered one of the most important criteria for teaching and learning effectiveness. In a study of seventh and eighth grade teachers, researchers found that more effective teachers devoted half of each period to combined lecture, demonstration, and discussion in contrast to less effective teachers who used a fourth of each period for these activities and half of the period for individual seatwork... In the same study, effective teachers asked more process questions (calling for explanations) and more product questions (calling for short answers) than less effective teachers. Level of student involvement was also found to be directly dependent upon the length of wait time—the time interval given for a response. In the average elementary school classroom, this time was one second before a question was redirected, restated or answered. By a simple increase in this time, the character of the classroom was changed to more interactive participation and detailed discussion by learners at every level of achievement. (Ohio Department of Education, 1983, p. 29)

Seventh and eighth grade mathematics is critical for the development of concepts that will enable the student to continue the study of mathematics. Therefore, when planning or developing a seventh and eighth grade mathematics program, educators must deal realistically with what is to be taught and the amount of time devoted to mathematics teaching.

APPENDIX A

There is concern that computation may be overstressed in the seventh and eighth grades with too little time being given to teaching concepts and problem-solving techniques.

> To many people the mathematics curriculum in kindergarten through eight is synonymous with computation. And in the broader context, the mathematics curriculum at all levels tends to be defined in terms of the many skills that are a part of the subject. While mathematics educators reject this limited viewpoint, it is the one that permeates the thinking of some educators and the public. (Trafton, 1980, p. 14)

Many believe that seventh and eighth grade students know **how** to add, subtract, multiply, and divide; however, test results indicate that few of them know **when** to add, subtract, multiply, or divide.

The assumption that the "cry" for back to basics meant more computation at the middle and upper grades was incorrect. The National Council of Supervisors of Mathematics (NCSM) has developed a position paper expressing this concern.

> ...basic skills must mean more than computation. The present technological society requires daily use of such skills as estimating, problem solving, interpreting data, organizing data, measuring, predicting, and applying mathematics to everyday situations. The changing needs of society, the explosion of the amount of quantitative data, and the availability of computers and calculators demand a redefining of the priorities for basic mathematics skills. (NCSM, 1977, p. 1)

To be an effective middle school mathematics teacher, one must be able to assist students in using elementary mathematical techniques. Seventh and eighth grade students should be able to use these techniques to solve problems. Students must be introduced to the processes of problem solving rather than continuing to focus on computational skill development.

APPENDIX A

Seventh and eighth grade teachers and teacher committees must determine how much of the students' time is going to be spent in review of the computational skills as opposed to the amount of time spent developing problem-solving techniques.

> Learning to solve problems is the principal reason for studying mathematics. Problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations. Solving word problems in texts is one form of problem solving, but students also should be faced with non-textbook problems. Problem-solving strategies involve posing questions, analyzing situations, translating results, illustrating results, drawing diagrams, and using trial and error. In solving problems, students need to be able to apply the rules of logic necessary to arrive at valid conclusions. They must be able to determine which facts are relevant. They should be unfearful of arriving at tentative conclusions, and they must be willing to subject these conclusions to scrutiny. (NCSM, 1977, p. 2)

The question, then, is not how much time should be devoted to the reteaching of computational skills, but how to use the computational skills already learned. Curriculum developers and mathematics teachers must grapple with both questions in preparing students to deal with future careers in our technological society.

APPENDIX B

CALCULATORS AND COMPUTERS

WHAT IS THE APPROPRIATE ROLE OF CALCULATORS?

Calculators may be used in seventh and eighth grade classrooms to aid students with problem-solving skills and computational weaknesses. Concerns regarding the possible effects of using calculators in the classroom continue among some parents and teachers. Although further research is needed, evidence to date suggests positive effects from certain uses of calculators in the classroom.

Suydam (as cited in Driscoll, n.d.), reviewing 100 research studies comparing noncalculator groups with groups that used calculators with some regularity in the classroom, found that few studies showed an advantage in mathematics achievement for the noncalculator groups. The majority of the studies revealed either no significant difference or an advantage for calculator groups.

The calculator, like any other instrument with potential benefits, is not immune to misuse. What, then, may be the legitimate role or use of calculators in the classroom?

Driscoll asserts that successful problem solvers have the ability to estimate, retrace solution steps, and question the reasonableness of answers.

> Two sixth grade classes, one using calculators and the other not, were the focus of the study. Each was provided regular instruction in the problem-solving skills. The study found that students using calculators applied those skills more frequently. (Driscoll, n.d., p. 114)

The use of calculators can assist students with computational weaknesses and promote a more meaningful and substantive mathematics program. Low-achieving students may benefit from the use of calculators as they concentrate on the process of problem solving. A study of sixth grade students found that the ability to determine which computational procedures were applicable to the solution of a given set of problems was improved among average-and low-ability students using calculators as opposed to comparison students who did not use calculators.

In conclusion, evidence suggests that calculators may be used in the seventh and eighth grade classrooms to enhance problem-solving skills such as estimating, retracing processes, and questioning the reasonableness of answers. Further, the use of calculators may ease the burden for students with computational weaknesses, enabling them to focus more on the processes of problem solving while improving their skills.

APPENDIX B

WHAT IS THE APPROPRIATE ROLE OF COMPUTERS?

All aspects of our society, including schools, will continue to be influenced by technology. It is essential for teachers to take advantage of the educational potential of instructional technology... microcomputers. Computers are tools...educational aids. They are enormously versatile and may be used in the educational program in any number of imaginative and useful ways: to create simulations, to enhance problem-solving skills, to create graphics, to apply one or more specific skills in a game situation, to enhance word processing capabilities, to retrieve information, to serve as a tutor, and to provide opportunities to drill and practice mathematical skills.

Computers can represent a substantial and multifaceted educational resource. The appropriate use of microcomputers is discussed in detail in *Microcomputer Planning Guide* (n.d.) by the Ohio Department of Education.

Peterson (1984) has noted a caution in relation to the role of the computer in drill and practice.

> Drill and practice is the McDonald's of computer education—homogenized, sanitized, orderly, and omnipresent. Its critics claim it emphasizes piecemeal learning and rote memorization at the expense of conceptual learning and understanding. In its defense many educators insist that drill and practice is a valid use of computers and there are studies that indicate it does improve standardized test scores. Yet the computer is such a marvelously versatile device that using it primarily for drill and practice seems analogous to dining mainly at a fast-food joint when there are a dozen gourmet restaurants within walking distance down the street. (Peterson, 1984, p. 14)

Walker (1983) has identified many ways that the computer can contribute to education, including these: providing more varied sensory and conceptual modes, allowing learning to occur nearer to the speed of thought, tailoring the learning to the individual, encouraging more independent learning and providing better aids to understanding abstractions.

APPENDIX B

Seven problems or limitations associated with the use of computers in education have been identified by Walker:

1. Computers can only supplement conventional education; they cannot substitute for it.
2. Teachers prepared to use computers are in short supply.
3. New products and systems are being created and marketed in such profusion, with such speed, and with so little standardization that systematic, long-term planning is nearly impossible.
4. Good programs are scarce because creating them is difficult, time-consuming, and expensive.
5. As we are only beginning to understand how to use computers in education, it is easy to make mistakes and misuse them.
6. Programs that deal with subject matter involving judgment, intuition, improvisation, and creativity are difficult to produce.
7. Computers will not solve, and may aggravate, current problems confronting education such as equity, school finance, and divergent public expectations. (Walker, 1983, pp. 105-106)

Although computers have limitations, the National Council of Teachers of Mathematics has stated that computers should be integrated into the seventh and eighth grade curriculum.

> With the increasing availability of microcomputers at decreasing costs, it is imperative that schools play an active part in preparing students of the 1980s to live in a world in which more and more functions are being performed by computers. (NCTM, 1980, p. 8)

IN WHAT WAYS MIGHT THE COMPUTER BE USED IN MATHEMATICS?

A variety of examples might be cited; however, the following examples are representative of uses of the computer in mathematics.

> If a district has a sizeable number of elementary students who are having difficulty performing basic mathematics computations, microcomputers could be used to provide those students with extra practice in those skills. The microcomputer in this instance would be used for drill and practice in the mathematical fundamentals. In another example, a district may use the microcomputer to teach junior high school students elementary computer programming skills which the students may use to enhance their problem-solving skills. The expected outcomes in this example are better problem-solving skills and an understanding of elementary programming concepts. (Ohio Department of Education, n.d., *Microcomputer Planning Guide*, p. 5)

APPENDIX C

SEX EQUITY IN MATHEMATICS

WHAT IS THE NATURE OF SEX DIFFERENCES IN MATHEMATICS PERFORMANCE?

The second mathematics assessment of the National Assessment of Educational Progress (NAEP II) provides the most comprehensive look at sex-related differences in learning mathematics on a nationwide basis (Fennema and Carpenter, 1981)...No clear pattern of differences in achievement between girls and boys was apparent at ages 9 and 13. Girls tended to score slightly higher than boys on the less complex tasks, whereas boys tended to score slightly higher on the understanding and application exercises. At age 17, the average performance of boys exceeded that of girls at every complexity level.

NAEP II compared the achievement of 17-year-old girls and boys who reported that they had been enrolled in the same mathematics courses. For each course background grouping (Algebra 1, Geometry, etc.), boys' achievement exceeded that of girls. The magnitude of the difference increased consistently in relation to the amount of mathematics taken. In other words, the difference in performance between 17-year-old girls and boys was smallest for students who had taken first-year algebra. It was even greater for students who had taken geometry, and the trend continued through courses beyond second-year algebra. The achievement differences between girls and boys increased as the cognitive complexity of the items increased. There were smaller differences at the less complex levels and larger differences at the higher levels.

Other studies have confirmed these results (Armstrong, 1980; California Assessment Program, 1978). When a large sample of girls and boys is selected from many schools and given tests of mathematics performance, little or no differences in achievement are found in elementary school, but late high school differences are often found, with boys achieving at higher levels, particularly on content of some sophistication, like problem solving. Although many have hoped, and even hypothesized, that most of these differences could be explained by the fact that boys have elected more advanced mathematics courses than girls have, such has not proved to be true. Even when girls and boys are in the same mathematics classes, starting in middle school, girls, as a group, fall behind boys in the important mathematical learning.

APPENDIX C

One question that should be raised about these sex-related differences in mathematics achievement is whether they appear consistently. The answer is no. Sex-related differences in mathematics often do not appear within individual schools. For example, in the Fennema-Sherman (1978) studies, girls were achieving as well as boys in two of the four high schools and in three of the four middle schools that were examined. This reinforces the idea that one should be extremely cautious about generalizing sex-related differences in achievement to an individual. Certainly, many girls achieve at higher levels than many boys and vice versa. Larger variation in achievement is found between high-and low-achieving boys or high- and low-achieving girls (intrasex) than between boys and girls (intersex). (Fennema, 1982, pp. 11-12)

APPENDIX D

MATHEMATICALLY ABLE STUDENTS AND ABILITY GROUPING

WHO ARE THE MATHEMATICALLY ABLE STUDENTS?

Mathematically able students may or may not be "gifted" students. They may or may not be high achievers in mathematics. Typically, the mathematically able students master the standard curriculum in one-third of the time other children take. "The normal school curriculum calls for a ratio of 70/30 in time devoted to teaching the basic skills and time devoted to high cognitive learning (reasoning, drawing inferences, and reaching conclusions)." (Correll, 1978, p. 8.) A reverse of this ratio is needed for the able child.

WHAT DOES RESEARCH SAY ABOUT ABILITY GROUPING?

> Ability grouping continues to be a controversial subject among educators and the lay public. Questions have been raised concerning the possible negative social effects of segregating either the slow or the bright students. Opponents of ability grouping for the gifted argue that we may be creating an intellectual elite. Proponents of grouping suggest that such an arrangement permits gifted children to work among peers who will serve to stimulate and challenge each other intellectually. (Correll, 1978, p. 28)

Gray (1983) has summarized the research relative to affective effects of ability grouping. First, the favorable self-concept of the students assigned to the high-achievement groups is reinforced. Second, the unfavorable self-concept of the students assigned to the low-achievement groups is reinforced. Third, low self-concepts operate against motivation for scholastic achievement. Fourth, grouping deprives low achievers of the stimulation of higher achieving children as learning models and helpers. Ability grouping is useful when the information gained by testing and observation is the first step in a program of diagnosis and individual instruction. (Findley, 1975; Gray, 1983)

APPENDIX E

LEARNING DISABILITIES

WHAT IS A LEARNING DISABILITY?

Three of the more common learning disabilities that affect the performance of middle school students in mathematics are perceptual, memory, and integrative disabilities. To clarify these difficulties further, Bley and Thornton (1982) have divided them into visual and auditory deficits. When these difficulties are recognized in students who have been mainstreamed, both the teacher and the student need to be taught ways of dealing with, compensating for, and living with the handicap.

EXAMPLES OF LEARNING DISABILITIES AFFECTING PERFORMANCE IN MATHEMATICS

Type of Learning Disability	Visual Deficit	Auditory Deficit
PERCEPTUAL		
Figure-ground	May not finish all problems on page	Trouble hearing pattern in skip counting by 5s
	Difficulty seeing subtraction within a division problem	Difficulty attending in the classroom because of extraneous noise
	Difficulty reading multidigit numbers	
Discrimination	Difficulty associating operations sign with problem (abstract reasoning)	Cannot distinguish between tenth and ten
	Cannot discriminate among operational symbols	Cannot distinguish between thirty and thirteen
	Difficulty reading fractional numbers	

APPENDIX E

Spatial	Trouble writing on lined paper	Difficulty following directions using ordinal numbers
	Trouble noticing size differences in shapes	
	Trouble with fraction concept due to inability to note equal-sized parts	
	Difficulty writing decimals	
	Difficulty aligning numbers	
	Difficulty writing fractional numbers (may also be reversal)	
	Difficulty with ordinal numbers	
MEMORY		
Short Term	Difficulty copying problems from the board (may be spatial)	Difficulty with dictated assignments
	Trouble retaining newly presented material	Difficulty with oral drills
Long Term	Inability to retain basic facts or processes over a long period	
	Difficulty solving multioperation computation	
Sequential	Difficulty following through on multiplication problems	Cannot retain information in dictated word problems
	Difficulty following through on long division problems	
	Difficulty solving column-addition problems	
	Difficulty solving multistep word problems	

APPENDIX E

INTEGRATIVE		
Closure	Difficulty reading multidigit number (see figure-ground)	Difficulty continuing counting from within a sequence
	Difficulty with missing addends and missing factors	
	Inability to draw conclusions; therefore, trouble noticing and continuing patterns	
	Trouble continuing counting pattern from within a sequence	
	Difficulty with word problems	
Receptive Language	Difficulty with words that have multiple meaning	Difficulty writing numbers from dictation
	Difficulty relating word to meaning (may be spatial)	Difficulty relating word to meaning
Expressive Language	Much difficulty with rapid oral drill	Difficulty explaining why a problem is solved as it is
		Difficulty continuing counting
Abstract Reasoning	Inability to compare numbers using symbols	Inability to follow the logic or oral explanations that focus on the meaning or rationale of a new concept or skill
	Inability to solve word problems	
	Inability to understand patterning in counting	
	Difficulty with decimal concept	

(Bley and Thornton, 1982, pp. 21-22)

APPENDIX E

WHAT ARE SOME OF THE CONSEQUENCES OF MAINSTREAMING?

Teachers of mathematics are encouraged to become more knowledgeable about how to deal with the recognition of the instructional methods for learning disabled students.

> The number of learning disabled students who are being mainstreamed is increasing... The way these learning difficulties manifest themselves varies greatly from one student to the next. Consequently, what helps one child learn and retain basic mathematical concepts and skills may not help another. Consulting special educators in the school or district is important in this regard. These resource persons can help explain a student's specific learning deficits and suggest general techniques for dealing with them. (Bley and Thornton, 1982, p. 20)

WHAT ARE SOME GENERAL PRACTICES AND SPECIFIC STRATEGIES FOR WORKING WITH HANDICAPPED STUDENTS?

As a result of mainstreaming, there is a redefining of the roles for both the special education teacher and the regular classroom teacher. Whereas the special education teacher has traditionally provided instruction for children with developmental and emotional handicaps or learning disabilities, these responsibilities are now shared with regular classroom teachers. Special education teachers may function more as consultants to regular teachers and provide instruction for only those students who need intensive instruction for limited periods of time. Also, students with more severe handicaps, with little opportunity to interact with nonhandicapped peers, will be attending regular classrooms with increased frequency. (Hasazi, et al, 1979)

Specialists, such as speech and physical therapists, physicians, and psychologists, are becoming part of an educational team that also includes parents, administrators, classroom teachers, and special education teachers. In an effort to provide an effective educational program for the handicapped student, yearly goals must be set, and the manner of educational delivery must be approved by the team members.

It is the responsibility of the local educational agency to insure that each handicapped child receives an appropriate education. The federal government assists the LEA by requiring that a portion of the funds appropriated as a result of Public Law 94-142 be allocated specifically for inservice training of administrators and teachers in the local school district.

APPENDIX E

Bley and Thornton (1982) suggest some specific intervention techniques.

- *Boxes, circles, and lines* may be used for students with figure-ground difficulties.
  1. *Box* in the problem in the mathematics textbook itself. This makes the problems easy to find and the student can more easily separate the problems.
  2. *Circle* the number of the problem so the student can recognize the problem number from the problem itself.
  3. *Underline* important directions with heavy lines or colors.

- *Use color coding* as a way to provide:
  1. a way of focusing attention;
  2. a way of properly sequencing steps;
  3. increased ability to recall information;
  4. a way of identifying starting and stopping points;
  5. a cue to the appropriate response;
  6. increased ability to act independently;
  7. visual reinforcement for those with auditory difficulties; and,
  8. a way of organizing the visual image to aid those with visual memory difficulties.

- *Reduce writing tasks* for students that have spatial, motor, or perceptual deficits by:
  1. creating standard formats for worksheets and providing paper masks to blot out portions of a page;
  2. teaching students to box off areas of their paper before copying problems into the boxes;
  3. keeping directions and explanations at a minimum using key words to focus attention; and,
  4. assigning small sections at a time.
- *Provide kinesthetic reinforcement* by having the children finger trace, verbalize, and then write the problem.

APPENDIX E

- *Adjust, alter or reinforce textbook presentations* in order to teach students with problems in auditory discrimination, special organization or abstract reasoning; the teacher may have to teach decimal and common fractions together or teach decimal fractions first.

- *Use auditory cueing* by letting students close their eyes while listening or use a tape recorder for frequent reinforcement. This is especially helpful to students with visual-perceptual or memory disabilities.

- *Use patterns, charts or sample problems* for visual direction clues.

- *Break instruction into small segments.*

There are many other instructional or intervention techniques that may be used to assist students with learning disabilities. The important thing is to learn what is best for a particular student and use the technique.

APPENDIX F

TEACHER PREPARATION FOR THE MIDDLE GRADES

WHAT WOULD BE APPROPRIATE TEACHER PREPARATION FOR TEACHERS OF SEVENTH AND EIGHTH GRADE MATHEMATICS?

Haertel (1978) noted the need for coordination between our knowledge of the formal reasoning abilities of the early adolescent and the sequencing and teaching of middle grade mathematics and science. She developed a strong rationale for further study of the relationships between cognitive style and instructional practices.

> Accumulated research seems to indicate that the cognitive abilities available to an individual are different at different points in the life cycle and at different points in adolescence. These include the ability to represent (model), to classify, to form concepts, to memorize, to solve problems, and to reason. At the risk of overgeneralization, it can be noted that virtually all students at early adolescence seem to have attained what Piaget termed the concrete operational state, whereas formal abilities, when exhibited, are specific rather than general and are, perhaps, influenced by instruction. (Wheeler and Bezuszka, 1982, p.2)

The current teacher preparation and certification process provides either specialization in elementary school curriculum or in secondary school content area, but usually not in both.

> As a consequence, the middle grade mathematics teacher with a specialization in elementary education may have had from one to four semesters of mathematics or mathematics methods or an undergraduate major or minor in mathematics. These classroom teachers are specialists in the rather clear-cut mathematics programs of the elementary school or the senior high school but not in the mathematics program of the middle grades. They are not prepared to handle both a different treatment of familiar content and a treatment of content intrinsically satisfying to mathematicians. (Wheeler and Bezuska, 1982, p. 4)

On the other hand, many secondary certificated teachers do not have specific background for teaching basic mathematical skills to low-achieving students who lack motivation and have reading difficulties as well.

APPENDIX F

Historically, teacher preparation and certification patterns have not provided a model to produce effective teachers of seventh and eighth grade mathematics. If these patterns persist, then specific programs of inservice and staff development will need to address several major areas:

1. A broad understanding of mathematics aptitude and attitude differences in early adolescence.
2. A greater range of understanding of the cognitive functioning and sequencing of mathematical skills in adolescence and the appropriate experiences relative to their level of functioning.
3. A wider range of understanding and ability in teaching mathematical knowledge. This would encompass the present knowledge base of the elementary education specialist. It would provide remediation and analytical ability with the fundamental arithmetic and interpretive skills. In addition, it would provide sufficient background for an understanding of the relationships between algebra and geometry, functions and elementary probability and statistics. (National Science Foundation, 1983, p. 32)

The Mathematics Association of America Committee on Undergraduate Programs in Mathematics (1983) has recommended the following courses as appropriate preparation for seventh and eighth grade mathematics teachers.

Course 1: Fundamental Mathematical Concepts I

Course 2: Fundamental Mathematical Concepts II

Course 3: Geometry for Elementary and Middle School Teachers

Course 4: Algebra and Computing for Elementary and Middle School Teachers

Course 5: An Introduction to Calculus

Courses 6-9: Four more courses selected from the following:

(a) Mathematics Application

(b) Discrete Mathematics

(c) Probability and Statistics

(d) Number Theory

(e) History of Mathematics

For further information on the specific content of the courses, one should consult the report.

APPENDIX F

While content knowledge is one necessary condition for an effective teacher, it is not a sufficient condition. Recommendations on methods courses for teacher preparation are provided by the NCTM Guidelines for the Preparation of *Teachers of Mathematics* (1983). These guidelines describe essential content for the "how" of teaching mathematics. The quality of teacher preparation in mathematics can be addressed if the current issues are confronted and honest efforts made to resolve the dilemmas.

APPENDIX G

SOLUTIONS TO PROBLEMS

Problems: Variables and Literal Expressions

1. a. $25 + u$
   b. the price decreased
2. a. $x + y$
   b. yes
3. $40 (\$8.65) + (H - 40)(1.5)(\$8.65)$
4. $b + N \cdot a$
5. $120/h$

Problems: Equations and Inequalities

1. 250.6 F
2. a. $4a + 3b + 2c + 1d + 0 \cdot e$
   b. $\dfrac{4a + 3b + 2c + 1d + 0 \cdot e}{a + b + c + d + e}$
3. Let u = number of miles driven in the city
   r = number of miles driven in the country
   Any combination of city and country driving such that
   $\frac{1}{16}\ u + \frac{1}{22}\ r \leq 20$
4. If x = dollar amount in sales, then the question is, when is $.08x > .05x + 51$. \$1700 per week in sales satisfies the conditions.
5. x = distance driven such that

$$.127x + 2500 = .271x$$
$$2500 = .144x$$
$$x \approx 17{,}361$$

Problems: Geometry

1. a. 214 cm
   b. circumference about 673 cm; approximately 10 people
2. a. Distance if going around perimeter is 2km. The diagonal distance is $\sqrt{2}$ km, therefore, the distance saved would be $2\text{km} - \sqrt{2}\text{ km}$
   b. $2\text{ km} - \sqrt{2}\text{ km} \approx .59$ km
   .59 km x 12 minutes/km ≈7 minutes

APPENDIX G

3. Measure a 3-unit segment from the corner on one side, and measure a 4-unit segment from the corner on the other side. Using knowledge of the Pythagorean relationship, it must be that if the angle is 90° then the distance between the endpoints is 5 units. If it is not, the angle is not a right angle.
4. a. D = (23, -7), E = (23, -22), F = (9, -22), G = (9, -20), H = (0, -20)
   b. It is possible to avoid negative coordinates if H is placed at (0, 2) and F is at (9, 0).
5. a. 12-foot roll will be $4 cheaper
   b. Same
   c. 12-foot roll is cheaper when $w < \frac{3}{4} l$ and more expensive when $w > \frac{3}{4} l$; given $w \leq 9 \leq l \leq 12$
   d. It is never cheaper

Problems: Measurement

1. A little more than $4,400 (The formula for surface area of a sphere is $SA = 4 \pi r^2$)
2. Answers will vary
3. 800 square cm of main color
   400 square cm of secondary color
   200 square cm each of remaining two colors
4. A standard can is just under 5 inches tall and has a diameter of just under 2 ½ inches. Therefore, the approximate volume of one can is
   $v = \pi (1.25)^2 (5)$ or about 24.5 cubic inches

   One million cans would have a volume of 24,500,000 cubic inches or 14,178.2 cubic feet and would fill a room approximately 20 feet by 70 feet by 10 feet.
5. Solution will vary depending on several things. For example, do either the carpet squares or rolls have patterns that must be matched? Must the roll always go in one direction, or can it be rolled so the weave is perpendicular? One answer: 300 squares
   24 feet and 1 inch of the roll.

Problems: Probability and Combinations

1. He is more likely to enter B.
   $P(B) = (1/2)(1/3) + (1/3) + (1/3)(1/3)$
   $= 11/18$
   $P(A) = (1/2)(1/3) + (1/3)(2/3)$
   $= 7/18$

APPENDIX G

2. 2 to 1

   If a red face is showing it means the card which is white on both faces has been eliminated. There are three faces that it could be, two of these are red and one is white; hence the odds are 2 to 1.

3. 1/6 x 1/6 x 1/6 = 1/216

4. a. .40 x .40 x .36 x .36 = .021
   b. 2(.40)(.60)(.36)(.36) + 2(.40)(.40)(.36)(.64) = .136
   c. (.40)(.36) = .144
   d. (.60)(.60)(.64)(.64) = .145
   e. 1 -.145 = .855
   f. The events are not mutually exclusive; for example, e) includes a), b), and c).

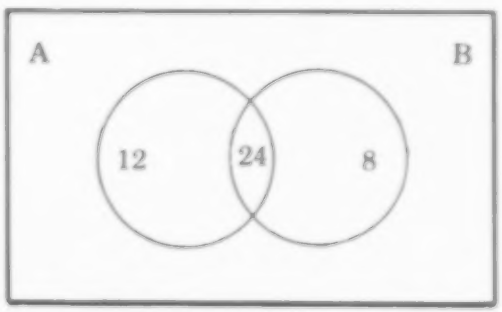
5. a. (.40)(.76) = .304
   b. (.40)(.50) = .20

6. Comment: A calculator or computer should be used if students are expected to carry out calculations. The first part of the problem may be too difficult for seventh and eighth grade students. However, it can be used to generate discussion.

   a. $(.95)^{12} \approx .54$
   b. $\binom{12}{9}(.95)^{9}(.05)^{3}+\binom{12}{10}(.95)^{10}(.05)^{2}+\binom{12}{11}(.95)^{11}(.05)^{1}+\binom{12}{12}(.95)^{12}(.05)^{0} \approx .998$

Problems: Problem Solving

1. There is insufficient information given to answer the problem. It provides a good opportunity for discussing problem solving.
2. Since 1/2 + 1/3 + 1/9 ≯ 1, the sons would never have been able to equitably divide the cows. However, 1/2 + 1/3 + 1/9 = 17/18 and that is why the loaned cow appeared to solve the dilemma.
3. The situation can be represented using a Venn Diagram.



APPENDIX G

One way to look at the problem is to say that reader A found 24 of the sample of 32 total errors found by reader B. That would give reader A a "success" rate of 24/32 or 3/4. It can be estimated that A would find 3/4 of the errors in any sample.

Since A found 36 errors, it follows that

$$3/4\ n = 36$$
$$n = 48$$

If there are 48 total errors and 44 (12 + 24 + 8) were detected by A and B, it would suggest that there are 4 undetected errors.

Problems: Applications

1. East Division
   St. Louis
   New York
   Montreal
   Philadelphia
   Chicago
   Pittsburgh

   West Division
   Los Angeles
   Cincinnati
   Houston
   San Diego
   Atlanta
   San Francisco

   Combined Ranking
   E St. Louis
   E New York
   W Los Angeles
   W Cincinnati
   E Montreal
   W Houston
   W San Diego
   E Philadelphia
   E Chicago
   W Atlanta
   W San Francisco
   E Pittsburgh

   No, though the first two teams are from the eastern division there does not appear to be a great imbalance between divisions.

2. Order 1,800 hot dogs
   Order 1,200 hamburgers
   Order 4,000 soft drinks

   Answers are approximate. Students should discuss whether it is better to "run out" or have leftovers. They should also consider if other factors, such as weather, can be considered.

3. 21.3 miles per hour

APPENDIX G

4. 1000m/14.9 seconds = $\dfrac{1000\text{m} \div (1000\text{m/km})}{14.9\text{ sec} \div (3600\text{ sec/hour})}$

$$= \frac{\dfrac{1000\text{m}}{1} \times \dfrac{1\text{ km}}{1000\text{ m}}}{\dfrac{14.9\text{ sec}}{1} \times \dfrac{1\text{ hour}}{3600\text{ sec}}}$$

$$= \frac{.1\text{ km}}{.0041\text{ hr}} = 24.4\text{ km/hr}$$

5. a. Eight 12-inch pizzas would be needed to serve 18 people
   b. No

6. $A = (\pi)(7^2)^2 - (\pi)(4^2)^2$
   $= 153.86 - 50.24$
   $= 103.62$ sq feet
   103.62 square feet $\approx$ 11.5 square yards
   4 inches = 1/9 yard
   11.5 square yards x 1/9 yard $\approx$ 1.3 cu yards
   1.3 cu yards @ 28.75 = $37.38

Problems: Functions and Relations

1.

	Input Number	Output Number
a.		15
b.		23
c.		31
d.	16	
e.	20	
f.	34	

2. a. He reached the 100 m distance 3 s units before his son
   b. About 70 m
   c. About 9 1/2 s

3. 2 kg + (.2 kg) (n)

APPENDIX G

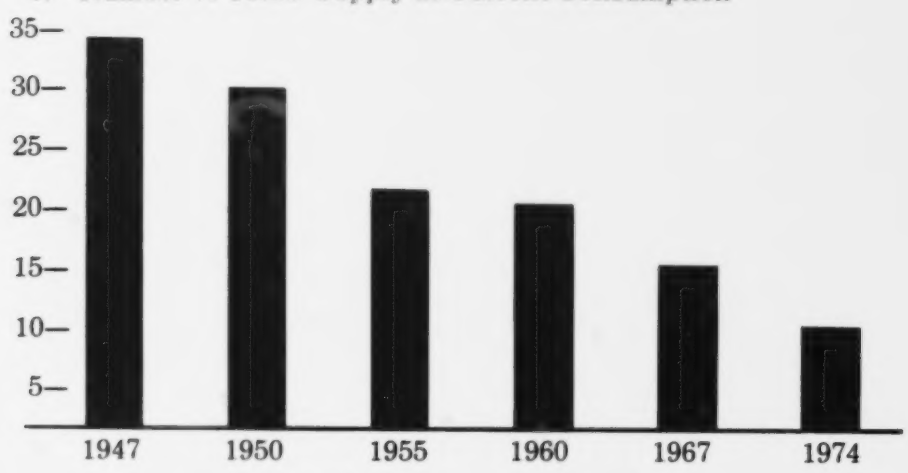
Problems: Statistics

1. Answers will vary
2. Students should discuss how much of the table is necessary to complete in order to determine trends. Some examples:
   a.

TABLE 1
Trends in U.S. Gas Production and Reserves
(In Trillions of Cubic Feet)

Year	(1) Additions to Reserve	(2) Production (Consumption)	(3) Difference between Additions and Productions (1) — (2)	(4) Proved Reserves	(5) Number of Years Supply of Gas Will Last (4) — (2)
1947	10	5	5	163	32.6
-					
-					
1950	12	6	6	183	30.5
-					
-					
1955	22	10	+12	220	22.0
1956	24	11	+13	233	21.2
-					
-					
1960	15	13	2	260	20
-					
-					
1967	22	18	+4	285	15.8
1968	13	19	-6	279	14.7
-					
-					
1973	9.0	22	-13	218	9.9
1974	8.7	21.6	-12.9	205.4	9.5

b. "Number of Years' Supply at Current Consumption"



(Only selected years are given)

APPENDIX G

c. Perhaps the data for the last ten years is available from your local gas company. If not, check with your local library.

3. Mean 4′ 5 4/5″
Median is 4′8″
Mode is 4′
Range is 11″

Problems: Technology

1.
$$\begin{array}{r} 2033 \\ \times\ 2033 \\ \hline 4{,}133{,}089 \end{array}$$

2. -4, 3, 5

Students can use a variety of techniques to solve this problem. Perhaps the simplest is guess and check.

3. x = no. of student tickets at $ .75
y = no. of adult tickets at $1.25

$x + y = 195 \Rightarrow x = 195 - y$
$.75x + 1.25y = 201.25$
$.75 (195-y) + 1.25y + 201.25$
$146.25 - .75y + 1.25y + 201.25$
$.50y = 55$
$y = 110$
$x = 85$

For seventh and eighth graders, perhaps the most appropriate method of solution is to write a program.

4.

+	42	1	93
15	57	16	108
37	79	38	130
16	58	17	109

Problems: Numbers and Numeration

1. $\sqrt{2}$ times or 1.4 times

2. a. Between 17 and 18 years
b. About 7.3 percent

3. Wife will get $3,200
Son will get $1,400
Daughter will get $1400

NOTES

LEADERSHIP SERIES

Citizenship, Multicultural and Human Relations Education

Conducting Self-Evaluation

Energy and Resource Conservation

Foreign Languages K-12

Intervention Assistance Teams

English Language Arts Curriculum

Integrating Language Arts

Developing Writing Competence

Mathematics 7-8

Ohio Studies

Quality Library Services K-12

For Additional Copies Write To:

The Ohio Department of Education
Division of Elementary and Secondary Education
65 South Front Street
Columbus, Ohio 43215
614/466-2761